OPTIMAL PORTFOLIO COMPOSITION
FOR 9 RISKY AND ONE RISK-FREE ASSETS

ECONOMICS OF FINANCIAL MARKETS

ECON 5520M

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Introduction

This paper is numerical solution for optimal portfolio choice by the mean-variance approach. Approach attempts to maximize portfolio expected return for a fixed amount of portfolio risk, or equivalently minimize risk for a constant level of expected return, by thoroughly selecting the proportions of different assets. Every combination of the risky assets, without including any holdings of the risk-free asset, can be plotted in risk-expected return space. Ensuing hyperbola forms the mean-variance frontier. The mean-variance frontier of a given set of assets is the boundary of the set of means and variances of the returns on all portfolios of the given assets. Many asset pricing propositions (e.g. CAPM) and test statistics are defined in terms of the mean-variance frontier. Figure 1 displays a regular mean-variance frontier. It is conventional to distinguish the mean-variance frontier of all risky assets, graphed as the hyperbolic region, and the mean-variance frontier of all assets, i.e., a risk-free rate, which is the larger wedge-shaped region. Some authors reserve the terminology ‘‘mean-variance frontier’’ for the upper fragment, calling the whole graph the minimum variance frontier. The risky asset frontier lies between two asymptotes, shown as dotted lines. The risk-free rate conventionally is below the intersection of the asymptotes and the vertical axis, or the point of minimum variance on the risky frontier. If it were above this point, investors with a mean-variance objective would try to short the risky assets, which cannot represent an equilibrium (we would actually observe it in this paper in the case of 100 stocks at a high rate of risk aversion). In general, portfolios of two assets fill out a hyperbolic curve through the two assets. The curve is sharper the less correlated are the two assets, because diversification make portfolio more efficient in mean-variance sense. Portfolios of a risky asset and risk-free rate give rise to straight lines in mean-standard deviation space.

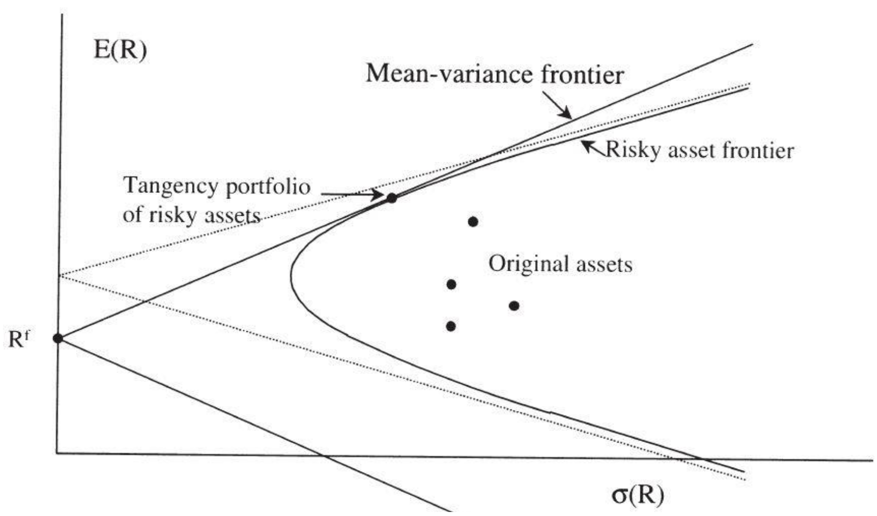

Figure 1: Mean-variance frontier
Source: John H. Cochrane. Asset pricing. p. 81. Princeton University Press, 2005

Data description

	Ticker
Capitalization range: $7.737-$363.101, mil	
PIMCO New York Municipal Income Fund	PNF
American Strategic Income Portfolio III	CSP
Dow 30 Premium	DPD
Capitalization range: $1125.950-$3026.322, mil	
Glimcher Realty Trust	GRT
Newcastle Investment Corporation	NCT
Industrias Bachoco, S.A. de C.V.	IBA
Capitalization range: $3037.480-$430299.000, mil	
Oracle Corporation	ORCL
MetLife, Inc.	MET
Enersis S A	ENI

Table 1: Stocks used in calculation within given capitalization ranges
Source: NYSE

Table 1 gives ticker interpretation. Table 2 contains a sample statistic for nine listed stocks from NYSE. Sample period is from January 1, 2009 until December 31, 2013. Individual stocks are selected in the following manner. Three random tickets picked from first, third and fifth quintiles of stocks ranged by market capitalization. Then the prices of these stocks examined on daily, weekly and monthly horizons. In addition to the four first sample moments, the table reports Jarque-Bera Statistic. Jarque-Bera statistics marked with asterisks are not statistically different from zero for $\alpha=0.05$, implying normality of given returns. None of stocks and indexes daily and weekly returns exhibit normality. Meanwhile, four stocks were normally distributed monthly returns. The net returns of IBA for both weekly and monthly frequencies had a moderate right skewness, so only log returns were normally distributed as JB indicates. Table 2 reports both log returns along with net returns because of testing for random walk 3 (RW3) hypothesis. Log returns are required to maintain limited liability principle. The basic test for random walk employed in this paper is variance ratio (VR) test. RW3 assumes that increments of the level of the random walk are uncorrelated at all leads and lags. Therefore, we may test it by testing the null hypothesis that the autocorrelation coefficients of the first-differences at various lags are zero. Variance ratio test can identify if the variance in time-series grows faster or slower than linearly. Formulas for optimal weights could be found by using brute-force Lagrangian approach. Out data is not normally distributed that is why we have to use quadratic utility function. With quadratic utility function, the higher order terms in Taylor approximation disappear. The limits of quadratic function is that it does not work for all levels of wealth. For the purpose of this work, the wealth is normalized to 1.

Ticker	Mean		Standart Deviation		Skewness		Kurtosis		Jarque-Bera Statistic	
Daily Returns										
	Net	Log	Net	Log	Net	Log	Net	Log	Net	Log
PNF	0.001	0.001	0.011	0.011	0.13	0.03	7.61	7.44	1118.28	1034.25
CSP	0.000	0.000	0.008	0.009	-0.56	-0.74	15.33	15.83	8036.67	8747.89
DPD	0.001	0.000	0.014	0.014	-1.03	-1.24	12.11	13.32	4571.07	5910.44
GRT	0.002	0.001	0.041	0.040	1.73	0.99	16.97	13.87	10862.07	6393.64
NCT	0.005	0.002	0.068	0.065	2.47	0.95	24.02	17.18	24441.11	10721.64
IBA	0.001	0.001	0.024	0.023	2.25	1.08	46.56	31.79	100531.35	43685.55
ORCL	0.001	0.001	0.018	0.018	-0.21	-0.41	8.31	8.75	1485.18	1768.21
MET	0.001	0.000	0.032	0.032	0.30	-0.34	14.18	14.47	6565.59	6916.04
ENI	0.000	0.000	0.015	0.015	-1.03	-0.40	12.11	7.97	929.70	1328.98
Weekly Returns										
	Net	Log	Net	Log	Net	Log	Net	Log	Net	Log
PNF	0.003	0.003	0.026	0.026	0.85	0.50	11.49	10.14	819.35	566.48
CSP	0.001	0.001	0.017	0.017	-0.75	-0.88	6.59	6.90	165.01	199.99
DPD	0.003	0.002	0.030	0.030	-0.84	-1.11	7.75	8.31	276.56	361.00
GRT	0.010	0.006	0.091	0.087	1.54	0.54	11.68	8.70	925.65	367.29
NCT	0.021	0.012	0.158	0.131	3.82	1.72	27.23	13.95	7042.59	1437.67
IBA	0.005	0.004	0.046	0.046	-0.25	-0.53	5.00	5.56	46.22	84.20
ORCL	0.004	0.003	0.037	0.037	-0.18	-0.35	3.98	4.10	11.92	18.58
MET	0.004	0.002	0.066	0.066	0.80	-0.40	14.38	13.33	1441.72	1172.01
ENI	0.002	0.001	0.035	0.036	-0.07	-0.30	5.00	5.47	43.91	70.21
Monthly Returns										
	Net	Log	Net	Log	Net	Log	Net	Log	Net	Log
PNF	0.010	0.009	0.046	0.046	0.43	0.26	3.66	3.46	2.91*	1.19*
CSP	0.004	0.003	0.035	0.035	0.04	-0.16	4.86	4.75	8.69	7.95
DPD	0.012	0.010	0.051	0.051	-0.62	-0.86	4.46	4.83	9.16	15.66
GRT	0.044	0.034	0.151	0.135	1.72	0.71	9.62	6.14	139.41	29.73
NCT	0.093	0.058	0.315	0.241	2.64	0.90	12.44	7.04	292.41	48.97
IBA	0.024	0.021	0.085	0.082	0.62	0.22	4.66	4.24	10.74	4.36*
ORCL	0.018	0.015	0.077	0.076	0.08	-0.17	3.17	3.20	0.13*	0.39*
MET	0.019	0.013	0.111	0.114	-0.28	-0.94	4.54	6.11	6.67	32.97
ENI	0.006	0.004	0.073	0.073	0.19	-0.03	3.11	2.97	0.40*	0.01*

Table 2: Sample statistics
Asterisks indicate JB statistics that are not statistically different from 0 at 5% level of significance

Lagrangian characterization of mean-variance frontier

The optimal weights are given by

$$w_{p^*i} = \left(\frac{\theta_k^{-1}}{2}\right)\sum_{j=1}^{N} v_{ij}\left(E\left[z_j\right]-r_f\right),\ i=1,...,N,\ \text{and}\ w_{p^*0} = 1-\left(\frac{\theta_k^{-1}}{2}\right)\left(a-cr_f\right), \tag{1}$$

where z_j $(j = 1, ..., N)$ and r_f are net returns.
For this optimal portfolio,

$$E\left[z_{p^*}\right] = r_f + \left(\frac{\theta_k^{-1}}{2}\right)\left(cr_f^2 - 2ar_f + b\right), \tag{2}$$

$$\sigma_{z_{p^*}} = \left(\frac{\theta_k^{-1}}{2}\right)^2\left(cr_f^2 - 2ar_f + b\right), \tag{3}$$

where

$$a = \sum_{i=1}^{N}\sum_{j=1}^{N} v_{ij}E\left[z_j\right],\ b = \sum_{i=1}^{N}\sum_{j=1}^{N} v_{ij}E\left[z_i\right]E\left[z_j\right],\ c = \sum_{i=1}^{N}\sum_{j=1}^{N} v_{ij},\ \theta_k^{-1} = -\frac{\partial V_k(\cdot)/\partial E\left[z_p\right]}{\partial V_k(\cdot)/\partial\sigma_{z_p}^2}, \tag{4}$$

and v_{ij} represent the elements of V^{-1}.

$W_k = W_0\left(1+z_p\right)$, $\sigma_{W_k}^2 = W_0^2\sigma_{z_p}^2$, $E\left[W_k\right] = W_0\left(1+E\left[z_p\right]\right)$.
For a quadratic utility function $u(W) = -\frac{1}{2}(y-xW)^2$,

$$\begin{aligned} E\left[u\left(W_k\right)\right] = V_k(E\left[z_p\right],\sigma_{z_p}^2) &= -\frac{1}{2}y^2 - \frac{1}{2}x^2\sigma_{W_k}^2 - \frac{1}{2}x^2\left(E\left[W_k\right]\right)^2 + yxE\left[W_k\right] \\ &= -\frac{1}{2}y^2 - \frac{1}{2}x^2W_0^2\sigma_{z_p}^2 - \frac{1}{2}x^2\left(W_0\left(1+E\left[z_p\right]\right)\right)^2 + yxW_0\left(1+E\left[z_p\right]\right). \end{aligned} \tag{5}$$

$$\frac{\partial V_k(E\left[z_p\right],\sigma_{z_p}^2)}{\partial E\left[z_p\right]} = -x^2W_0^2\left(1+E\left[z_p\right]\right) + yxW_0 = xW_0\left(y - xW_0\left(1+E\left[z_p\right]\right)\right), \tag{6}$$

$y - xW_0\left(1+E\left[z_p\right]\right) > 0$ because $y - xW > 0$ for any W and then in expectation, $E\left[W_k\right]$,

$$\frac{\partial V_k(E\left[z_p\right],\sigma_{z_p}^2)}{\partial\sigma_{z_p}^2} = -\frac{1}{2}x^2W_0^2. \tag{7}$$

For simplicity, normalize W_0 to 1. Then,

$$\frac{\partial V_k(E\left[z_p\right],\sigma_{z_p}^2)}{\partial E\left[z_p\right]} = x\left(y - x\left(1+E\left[z_p\right]\right)\right), \tag{8}$$

$$\frac{\partial V_k(E\left[z_p\right],\sigma_{z_p}^2)}{\partial\sigma_{z_p}^2} = -\frac{1}{2}x^2, \tag{9}$$

$$\theta_k^{-1} = -\frac{\partial V_k(E\left[z_p\right],\sigma_{z_p}^2)/\partial E\left[z_p\right]}{\partial V_k(E\left[z_p\right],\sigma_{z_p}^2)/\partial\sigma_{z_p}^2} = -\frac{x\left(y - x\left(1+E\left[z_p\right]\right)\right)}{-(1/2)\,x^2} = \frac{2\left(y - x\left(1+E\left[z_p\right]\right)\right)}{x} > 0. \tag{10}$$

Take some values of y and x ($x = 1$, e.g.) and substitute θ_k^{-1} from (10) (that is true for any portfolio p including p^*) into (2), solve (2) for $E\left[z_{p^*}\right]$.
After finding $E\left[z_{p^*}\right]$, compute θ_k^{-1} in (10) and then w_{p^*i} and w_{p^*0} in (1) and $\sigma_{z_{p^*}}$ in (3), as well as

$$RRA\left(y,x,E\left[z_{p^*}\right]\right) = \frac{x}{y - xW_0\left(1+E\left[z_{p^*}\right]\right)}W_0\left(1+E\left[z_{p^*}\right]\right) = \frac{x\left(1+E\left[z_{p^*}\right]\right)}{y - x\left(1+E\left[z_{p^*}\right]\right)} > 0. \tag{11}$$

Hence, for each pair of y and $x = 1$, we have the associated portfolio and $RRA\left(y,x,E\left[z_{p^*}\right]\right)$. Choose the portfolio that corresponds to your $RRA\left(y,x,E\left[z_{p^*}\right]\right)$.

Optimal portfolio of nine stocks and a risk-free asset under the assumption of random walk

Daily frequency

Before reporting the optimal weight and discuss the results we need to pick a coefficient of relative risk aversion (RRA). RRA is a function of y, x and expected return of portfolio with optimal weights. X is normalized to one, while expected returns is determined endogenously. Hence, by changing variable y we can characterized RRA to differentiate investors given their attitude to risk.

$E[Z_p]$	$\sigma[Z_p]$	Sharpe ratio	RRA	Y
0.0174	0.0032	5.438	5.570	1.2
0.0260	0.0071	3.662	3.745	1.3
0.0346	0.0126	2.746	2.832	1.4
0.0433	0.0197	2.198	2.284	1.5
0.0519	0.0284	1.827	1.919	1.6
0.0606	0.0387	1.566	1.659	1.7
0.0692	0.0505	1.370	1.463	1.8
0.0779	0.0640	1.217	1.311	1.9
0.0865	0.0790	1.095	1.189	2.0
0.0952	0.0956	0.996	1.090	2.1
0.1038	0.1137	0.913	1.007	2.2
0.1125	0.1335	0.843	0.937	2.3
0.1211	0.1548	0.782	0.877	2.4
0.1297	0.1777	0.730	0.825	2.5
0.1384	0.2022	0.684	0.779	2.6

Table 3: Coefficients of relative risk aversion for daily frequency

Important point here is that RRA is a relative characteristic not absolute, so that exact values for RRA and Y are not important. We assume that our RRA is 2, or 1.919. That corresponds to Sharp ratio of 1.827, which is quite realistic for typical daily trading. This attitude to risk gives the following weights for assets.

CSP	-19.5
DPD	8.0268
ENI	-4.901
GRT	4.8544
IBA	-13.3
MET	8.2058
NCT	5.0216
ORCL	9.9443
PNF	25.814
Risk Free:	-23.2

Table 4: optimal weight for 9 risky and one risk-free assets on daily frequency with RRA 0.0519

As reported before the return for portfolio with this RRA is 0.0519, while standard deviation is 0.0284. The interpretation of the weights is the following. Negative weight on risk free assets means that we borrow at risk-free rate 23.2 times our initial wealth, or initial investment. Negative weight on risky assets indicate that we sell it short: CSP is sold short 19.5 times the initial investment and we do the same for ENI, 4.901 and IBA, 13.3 all times the initial investment. Positive weight on risky assets indicate buying: DPD must be bought 8.0268 times the initial investment, while GRT, 4.8544, MET, 8.2058, NCT, 5.0216, ORCL 9.9443 and PNF 25.814.

Weekly frequency

$E[Z_p]$	$\sigma[Z_p]$	Sharpe ratio	RRA	Y
0.1700	0.0560	3.036	3.546	1.5
0.2379	0.1098	2.167	2.679	1.7
0.3059	0.1815	1.685	2.198	1.9
0.3738	0.2712	1.378	1.892	2.1
0.4417	0.3788	1.166	1.680	2.3
0.5096	0.5044	1.010	1.524	2.5
0.5776	0.6479	0.891	1.406	2.7
0.6455	0.8094	0.798	1.312	2.9
0.7134	0.9888	0.721	1.236	3.1
0.7813	1.1861	0.659	1.173	3.3
0.8492	1.4014	0.606	1.120	3.5
0.9172	1.6346	0.561	1.075	3.7
0.9851	1.8857	0.522	1.037	3.9
1.0530	2.1548	0.489	1.003	4.1
1.1209	2.4419	0.459	0.973	4.3

Table 5: Coefficients of relative risk aversion for weekly frequency

We assume RRA 2.679.

CSP	1.6043
DPD	13.4282
ENI	-11.707
GRT	5.2408
IBA	-16.3065
MET	-12.6804
NCT	4.173
ORCL	16.8723
PNF	11.8223
Risk Free:	-11.4471

Table 6: Optimal weight for 9 risky and one risk-free assets on weekly frequency with RRA 2.679

This weight distribution gives 0.2379 of returns with 0.1098 as standard deviation. As this weight distribution consist of borrowing 11.44 times the initial investment. Investment then further extended by selling short ENI, 11.7, IBA, 16.3 and MET, 12.7 times the initial wealth. Positive weights indicate buying: CSP, 1.6, DPD, 13.42, GRT, 5.24, NCT, 4.2, ORCL 16.82 and PNF, 11.82 times the initial investment.

Monthly frequency

$E[Z_p]$	$\sigma[Z_p]$	Sharpe ratio	RRA	Y
0.3971	0.0407	9.757	13.582	1.5
1.1908	0.3677	3.239	7.086	2.5
1.9845	1.0223	1.941	5.790	3.5
2.7782	2.0043	1.386	5.235	4.5
3.5719	3.3138	1.078	4.926	5.5
4.3656	4.9508	0.882	4.730	6.5
5.1593	6.9152	0.746	4.594	7.5
5.9530	9.2072	0.647	4.494	8.5
6.7467	11.8267	0.570	4.418	9.5
7.5404	14.7736	0.510	4.358	10.5
8.3341	18.0481	0.462	4.310	11.5
9.1277	21.6500	0.422	4.269	12.5
9.9214	25.5794	0.388	4.236	13.5
10.7151	29.8363	0.359	4.207	14.5
11.5088	34.4207	0.334	4.182	15.5

Table 7: Coefficients of relative risk aversion for monthly frequency

We assume RRA 5.79 since that coefficient gives reasonable Sharpe ratio of 1.941.

CSP	25.2631
DPD	96.0683
ENI	-61.9183
GRT	20.665
IBA	-17.058
MET	-30.3994
NCT	6.7155
ORCL	10.6992
PNF	16.637
Risk Free:	-65.6723

Table 8: Optimal weight for 9 risky and one risk-free assets on monthly frequency with RRA 5.79

These weights give 1.198 of expected return with 1.0223 of standard deviation. Investment capacity is extended by borrowing at risk-free rate 65.7 times the initial wealth, while selling CSP 25, DPD 97, GRT 20.7, NCT 6.7, ORCL 10.7 and PNF 16.7 times the initial wealth.

Optimal portfolio of nine stocks and a risk-free asset based on random walk test results

Ticker	Mean	*Predictability*		
		First lag	*Second lag*	*Third lag*
		Daily Returns		
PNF	0.0005	0.0543	0.0224	-0.0468
CSP	0.0001	-0.0102	-0.0014	-0.1006
DPD	0.0004	-0.0023	-0.0493	-0.0029
GRT	0.0013	-0.1132	-0.0348	0.0138
NCT	0.0024	-0.2062*	0.1256	-0.1055
IBA	0.0009	-0.0515	-0.0630	0.0210
ORCL	0.0006	-0.0436	0.0370	-0.0314
MET	0.0004	-0.1422*	0.1306	-0.1194
ENI	0.0002	0.0575	0.0017	-0.0623
SPY	0.0006	-0.0764	0.0306	-0.0637
		Weekly Returns		
PNF	0.0026	-0.0868	0.0564	-0.0259
CSP	0.0007	-0.0637	0.0678	0.0687
DPD	0.0021	-0.0049	-0.0997	0.0400
GRT	0.006	-0.146	0.0701	0.024
NCT	0.012	-0.140	0.028	-0.084
IBA	0.004	-0.045	0.012	0.077
ORCL	0.003	0.0656	-0.0293	-0.0452
MET	0.002	0.0106	0.0657	-0.1456
ENI	0.0011	-0.1033	0.0409	0.0474
SPY	0.003	-0.0568	0.0094	-0.0184
		Monthly Returns		
PNF	0.0090	0.1565	-0.0654	0.0409
CSP	0.0032	0.3338*	0.0951*	-0.1004
DPD	0.0102	-0.0149	-0.2406	-0.0927
GRT	0.0336	0.1137	0.1256	0.1072
NCT	0.0578	0.0761	0.0486	0.0142
IBA	0.0206	0.0167	0.0826	0.1196
ORCL	0.0147	-0.1515	-0.1669	0.0905
MET	0.0125	-0.0560	-0.0624	-0.0188
ENI	0.0061	-0.0387	0.0395	-0.1795
SPY	0.0154	-0.0679	-0.0819	0.0514

Table 9: Statistic for testing RW3
Only the first three lags are shown since there were no statistically significant results past the second lag
Asterisks indicate the autocorrelation coefficient statistically different from 0 at 5% level of significance under RW3 based on VR statistic. Sample size for daily returns 1258, weekly 262, and monthly 60

On daily horizon, based on variance ratio statistic we have two stocks with one significant lag each: NCT and MET. Value of VR for NCT is 0.7938 and normalized Psi value is -2.6937. Value of VR for MET is 0.8578 and normalized Psi value is -2.2974. Data also suggest that CSP on monthly frequency has two significant coefficients under RW3. Psi values statistics for rho 1 and rho 2 are 1.966 (VR = 1.3338) and 1.9658 (VR = 1.5084), respectively.

Daily frequency

$E[Z_p]$	$\sigma[Z_p]$	Sharpe ratio	RRA	Y
0.0164	0.0030	5.467	5.536	1.2
0.0246	0.0067	3.672	3.720	1.3
0.0327	0.0120	2.725	2.812	1.4
0.0409	0.0187	2.187	2.267	1.5
0.0490	0.0270	1.815	1.904	1.6
0.0572	0.0367	1.559	1.645	1.7
0.0654	0.0480	1.363	1.450	1.8
0.0735	0.0607	1.211	1.299	1.9
0.0817	0.0750	1.089	1.178	2.0
0.0899	0.0907	0.991	1.079	2.1
0.0980	0.1080	0.907	0.996	2.2
0.1062	0.1267	0.838	0.927	2.3
0.1144	0.1469	0.779	0.867	2.4
0.1225	0.1687	0.726	0.815	2.5
0.1307	0.1919	0.681	0.770	2.6

Table 10: Coefficients of relative risk aversion for daily frequency with random walk 3 test

Again, Sharpe ratio around 2 is the most realistic average return on a unit of risk. It corresponds to RRA 2.267

CSP	-16.439
DPD	6.0318
ENI	-3.9516
GRT	4.0659
IBA	-11.0845
MET	2.2384
NCT	4.6253
ORCL	8.0053
PNF	21.5517
Risk Free:	-14.0434

Table 11: Optimal weight for 9 risky and one risk-free assets on daily frequency with random walk 3 with RRA 2.267

Negative weight on risk-free assets means borrowing at risk-free rate. This optimal portfolio configuration assumes borrowing 14 times the initial wealth. CSP should be sold short 16.4 times, ENI 3.9 times and IBA 11 times the initial investment. The gained leverage is then invested in DPD

6.03 times the initial investment, while GRT 4.06, MET 2.23, NCT 4.62, ORCL 8 and PNF 21.5 times the initial wealth.

Weekly frequency

$E[Z_p]$	$\sigma[Z_p]$	Sharpe ratio	RRA	Y
0.1700	0.0560	3.036	3.546	1.5
0.2379	0.1098	2.167	2.679	1.7
0.3059	0.1815	1.685	2.198	1.9
0.3738	0.2712	1.378	1.892	2.1
0.4417	0.3788	1.166	1.680	2.3
0.5096	0.5044	1.010	1.524	2.5
0.5776	0.6479	0.891	1.406	2.7
0.6455	0.8094	0.798	1.312	2.9
0.7134	0.9888	0.721	1.236	3.1
0.7813	1.1861	0.659	1.173	3.3
0.8492	1.4014	0.606	1.120	3.5
0.9172	1.6346	0.561	1.075	3.7
0.9851	1.8857	0.522	1.037	3.9
1.0530	2.1548	0.489	1.003	4.1
1.1209	2.4419	0.459	0.973	4.3

Table 12: Coefficients of relative risk aversion for weekly frequency with random walk 3 test

We are reporting weights for RRA 2.676.

CSP	1.6043
DPD	13.4282
ENI	-11.707
GRT	5.2408
IBA	-16.3065
MET	-12.6804
NCT	4.173
ORCL	16.8723
PNF	11.8223
Risk Free:	-11.4471

Table 13: Optimal weight for 9 risky and one risk-free assets on weekly frequency with random walk 3 with RRA 2.676

We extend our initial investment by borrowing at the free-risk rate, negative weight on risk-free asset indicate this, and by selling short risky stocks, negative weight on risky assets is the indication of that. Since the wealth in our calculations is normalized to 1, we just have to multiply the values in the table by the wealth that we are factually going to have. Therefore, we are borrowing 11.44 times the wealth, and then sell ENI, IBA and MET. We then buy CSP 1.6 times and, DPD 13.42, GRT 5.2, NCT 4.2, ORCL 16.9 and PNF 11.8 all times the initial investment that we are going to have.

Monthly frequency

$E[Z_p]$	$\sigma[Z_p]$	Sharpe ratio	RRA	Y
0.4049	0.0384	10.544	14.765	1.5
1.2140	0.3468	3.501	7.742	2.5
2.0232	0.9640	2.099	6.341	3.5
2.8324	1.8901	1.499	5.740	4.5
3.6415	3.1249	1.165	5.407	5.5
4.4507	4.6686	0.953	5.195	6.5
5.2599	6.5212	0.807	5.048	7.5
6.0690	8.6825	0.699	4.940	8.5
6.8782	11.1527	0.617	4.858	9.5
7.6874	13.9317	0.552	4.793	10.5
8.4966	17.0196	0.499	4.740	11.5
9.3057	20.4163	0.456	4.697	12.5
10.1149	24.1218	0.419	4.660	13.5
10.9241	28.1361	0.388	4.629	14.5
11.7332	32.4593	0.361	4.602	15.5

Table 14: Coefficients of relative risk aversion for monthly frequency with random walk 3 test

For RRA 6.341 the weight are going to be

CSP	52.8569
DPD	94.9263
ENI	-56.1482
GRT	17.4609
IBA	-20.606
MET	-36.2688
NCT	5.4432
ORCL	14.5263
PNF	12.0231
Risk Free:	-83.2137

Table 15: Optimal weight for 9 risky and one risk-free assets on weekly frequency with random walk 3 with RRA 6.341

The weights here are especially unrealistic. That is only possible in no tax and no transaction costs environment. Here we have to borrow 83 time the initial investment, while selling short ENI 56 and MET 36 times, while buying CSP 52, DPD 94, GRT 17.5, NCT 5.4, ORCL 14.5 and PNF 12 times the initial investment.

Optimal portfolio of 100 stocks and a risk-free asset based on random walk test results

BWP	CAS	CEB	CHL	CIR
BWS	CAT	CEC	CHN	CI
BXC	CAT	CEE	CHS	CKP
BXM	CBB	CEL	CHT	CLB
BXP	CBD	CEO	CHU	CLC
BXS	CBG	CEQ	CIA	CLF
BX_	CBI	CE_	CIB	CLG
BYD	CBK	CFI	CIE	CLH
BYI	CBL	CFR	CIF	CLI
BYM	CBM	CFX	CIG	CL
BZH	CBR	CF	CII	CSP
CAB	CBS	CCK	CIM	C
CAC	CBT	CCL	CGA	DPD
CAE	CBU	CCO	CGG	ENI
CAF	CBZ	CCU	CGI	GRT
CAG	CB	CCZ	CHA	IBA
CAH	CCC	CDE	CHD	MET
CAJ	CCE	CDI	CHE	NCT
CAM	CCI	CDR	CHH	ORC
CAP	CCJ	CEA	CHK	PNF

Table 16: 100 stocks for the same code to test the weights convergence to something realistic
The table contains tickers of risky stocks that were used as input for the same code

The key idea is that if more stocks are used, the weights might converge to something more realistic. The results are reported selectively. For daily returns, we have to use RRA 6.01 for realistic information ratio (2.69). The weight for risk free asset reaches -1344, while for risky assets weights range from -100, for CAJ, to 319, for CAG. For weekly, with RRA of 57, weight for risk-free rate is -239.8751. The smallest weight for risky asset is -90, for DPD, while the highest is 91 for CI. For monthly returns, we get completely erratic results. On the interval of RRA from -20 to 5 all defined portfolios have 0 risk. The procedure assigns all weight to the risk-free asset, while the weights for risky assets are zero. As the result, we do not observe convergence with greater number of stocks.

Special interest represents the weekly frequency because we do not reject RW3 on our basic data on weekly returns. Table 17 contains that statistics for RRA 132 and y equal to 1.5. Interestingly, but efficiency of portfolio actually decreased. Expected returns fell down, while variance increased. It happens because in most cases regressing on significant lags decreases the predicted price, rather than increases it. While, variance is still kept stationary. Testing for RW is more realistic, yet less pleasant.

Ticker	RW	RW test	Ticker	RW	RW test	Ticker	RW	RW test
BWP	-4.2222	-4.5288	CB_	-0.0073	-0.1339	CHS	0.772	0.798
BWS	2.8207	2.8141	CCC	-1.0577	-1.2144	CHT	7.8569	7.8482
BXC (1)	-1.7714	-1.9575	CCE	5.7272	5.8841	CHU	-4.1857	-4.2801
BXM	-1.128	-1.0961	CCI	8.3072	8.5445	CIA (3)	-1.07	-1.1518
BXP*	1.7655	1.665	CCJ	-2.0493	-2.0919	CIB	5.6939	5.9344
BXS	0.5873	0.5055	CCK	4.407	4.449	CIE	2.9589	3.0064
BX_	0.543	0.7607	CCL	-4.8682	-4.9474	CIF	6.7475	6.7261
BYD	-0.1027	-0.089	CCO	-1.454	-1.4578	CIG	-1.8246	-1.7316
BYI	-1.9457	-1.9536	CCU	3.2703	3.1146	CII	-4.5127	-4.7335
BYM (2)	4.0336	4.3797	CCZ	1.6567	1.6139	CIM (1)	-1.539	-1.5263
BZH	1.0169	0.9519	CDE	-1.2358	-1.1215	CIR	-0.1605	-0.0728
CAB	-0.1426	-0.1347	CDI	-5.366	-5.5527	CI	13.739	13.8777
CAC	2.8255	2.8763	CDR	-1.5036	-1.4287	CKP	-4.3566	-4.4475
CAE	0.4393	0.2382	CEA	0.9881	1.0517	CLB	4.5365	4.5112
CAF	-0.5303	-0.6131	CEB	2.4463	2.4975	CLC (8)	0.0283	0.0571
CAG	-1.4545	-1.4686	CEC	0.3523	0.3245	CLF	-2.5935	-2.6656
CAH	2.5597	2.7087	CEE (6)	2.6953	2.2839	CLG	-2.3703	-2.3598
CAJ	-4.316	-4.2585	CEL (8)	-0.6363	-0.8909	CLH (8)	-0.5215	-0.5244
CAM (2)	2.5349	2.4714	CEO	2.4177	2.3851	CLI	-1.4196	-1.4631
CAP (2)	1.5131	1.482	CEQ	-2.4199	-2.3671	CL	10.4382	10.1277
CAS	2.1867	2.3346	CE	-2.9696	-3.1049	CSP	-11.727	-11.7575
CAT	-3.9447	-4.0101	CFI	2.1499	2.2833	C	0.0781	-0.018
CAT	-1.6232	-1.6412	CFR	4.4972	4.5134	DPD	-13.7938	-13.7391
CBB	1.6803	1.6484	CFX	4.6779	4.6852	ENI	-9.5104	-9.6027
CBD	-1.5313	-1.4731	CF	2.9195	2.9371	GRT	5.417	5.483
CBG	-0.6752	-0.7576	CGA	-0.6754	-0.6908	IBA	-3.5399	-3.6126
CBI	2.3153	2.3358	CGG	-3.2286	-3.2358	MET	-3.1519	-2.9742
CBK	-1.0035	-1.0827	CGI	2.9346	2.9778	NCT	0.0947	0.0951
CBL	-3.9397	-4.0249	CHA	2.4694	2.6223	ORC	4.6685	4.815
CBM	1.2363	1.2613	CHD (8)	7.7966	8.0365	PNF	1.2323	1.2286
CBR	-0.7799	-0.8454	CHE	2.9756	2.9022			
CBS	4.1282	4.1593	CHH (2)	4.3871	3.9704	RF:	-36.8803	-35.4761
CBT	-1.291	-1.3382	CHK	-2.6599	-2.8007	E[Zp]	0.4889	0.4888
CBU	3.9145	4.2245	CHL (8)	-0.582	-0.5494	σ[Zp]	0.0054	0.0055
CBZ (8)	-1.3742	-1.6697	CHN	-0.7914	-0.7359			

Table 17: Comparing the results under assumption of RW and test of RW for weekly frequency
A number next to ticker indicates a number of significant lags. RW3 is tested by VR

Conclusion

To compare results we inspect the computed optimal weights keeping the same frequency and value of Y

Ticker	Random walk	Testing for RW	Random walk	Testing for RW	Random walk	Testing for RW
	Monthly (y=3.5)		Weekly (y=1.7)		Daily (y=1.6)	
CSP	25.2631	52.8569*	1.6043	1.6043	-19.5000	-19.7272
DPD	96.0683	94.9263	13.4282	13.4282	8.0268	7.2383
ENI	-61.9183	-56.1482	-11.7070	-11.7070	-4.9010	-4.7420
GRT	20.6650	17.4609	5.2408	5.2408	4.8544	4.8792
IBA	-17.0580	-20.6060	-16.3065	-16.3065	-13.3000	-13.3016
MET	-30.3994	-36.2688	-12.6804	-12.6804	8.2058	2.6861*
NCT	6.7155	5.4432	4.1730	4.1730	5.0216	5.5504*
ORCL	10.6992	14.5263	16.8723	16.8723	9.9443	9.6066
PNF	16.6370	12.0231	11.8223	11.8223	25.8140	25.8626
Risk Free:	-65.6723	-83.2137	-11.4471	-11.4471	-23.2000	-17.0525
E[Zp]	1.9845	2.0232	0.2379	0.2379	0.0519	0.0490
σ[Zp]	1.0223	0.9640	0.1098	0.1098	0.0284	0.0270

Table 18: Comparing the results under assumption of RW and test of RW
Asterisks indicate that given security has autocorrelation coefficient(s) statistically different from 0 at 5% level of significance under RW3 based on VR statistic.

In monthly returns, CSP security has two significant lags. Both rho coefficient are positive (cf. table 9) meaning that positive change in past indicates ascending pricing for the period of prediction. The optimal weights algorithm assigned to CSP more than twice the amount of investment in comparison to optimal allocation under random walk hypothesis. That led to corresponding change in all optimal weight allocation, including risk-free asset. Eventually that configuration made up a more efficient portfolio that has higher returns with lower risk. In the data weekly returns did not have statistically significant lags, that the reason for identical results in weights, returns and standard deviation. Testing for random walk in daily returns actually slightly decreased the efficiency of portfolio. Two securities, MET and NCT, both have significant lags with negative value of rho coefficients. Algorithm decreased quadruply the investment in MET, while the investment in NCT is kept the same.

Predictability components in stocks does lead to change in optimal weight allocation, however, as data suggest it does not necessarily for good. Efficiency of portfolios might both increase and decrease. There is no apparent pattern. The weight allocation is quit theoretical as we already conceded. We operate in the tax and transaction costs free environment, frictionless world.

Normally there are limits on borrowing at risk-free rate and number of transactions are minimized especially in daily returns, where those costs are comparably to returns.

The computations for this paper are done in GAUSS. The code itself and output file are attached in appendix.

```
library pgraph;

screen off;

output file=asstB.res reset;
outwidth 256;
format /rd 10,4;
/*Sample Size: Daily=1258, Weekly=262, Monthly=60*/

lags=10;

y=filesa("*csv");
y_rows= ROWS(y);

daily_matrix=zeros((y_rows/3),7);
weekly_matrix=zeros((y_rows/3),7);
monthly_matrix=zeros((y_rows/3),7);
monthly_i=1;
weekly_i=1;
daily_i=1;

/*creating a matrix of returns for daily, weekly and monthly data*/
mor_d=zeros(1247,(y_rows/3));
mor_w=zeros(251,(y_rows/3));
mor_m=zeros(49,(y_rows/3));

file_count=1;
do until file_count>y_rows;
file_name=y[file_count,1];

/*loading the data set as a single string*/
load data[]= ^file_name;
T= rows(data)/8-1;
/*Transforming the single string data set into a proper (T+1) by 8 data set*/
data1=reshape(data,(T+1),8);
/*Assigning the 'adjusted close' data - the data to be worked with*/
adjclose= data1[(T+1):2,8];
/*Fixing the Irrational number error when loading raw data from Yahoo finance*/
adjclose=abs(adjclose);
logclose=ln(adjclose);
/*net and log of return #ofreturns x 1 matrices*/
net_ret=(adjclose[2:T,1]-adjclose[1:(T-1),1])./adjclose[1:(T-1),1];
log_ret=logclose[2:T,1]-logclose[1:(T-1),1];

/*Means, Variance, Skewness, Kurtosis for Normality Test*/
mean=meanc(net_ret);
variance=vcx(net_ret);
log_mean=meanc(log_ret);
log_variance=vcx(log_ret);
sumcubed=0;
sumquad=0;
logsumcubed=0;
logsumquad=0;

i=1;
do until i>(T-1);
      sumcubed=sumcubed+(net_ret[i,.]-mean)^3;
      sumquad=sumquad+(net_ret[i,.]-mean)^4;
      logsumcubed=logsumcubed+(log_ret[i,.]-log_mean)^3;
      logsumquad=logsumquad+(log_ret[i,.]-log_mean)^4;
```

```
i=i+1;
endo;

/*Net Results - Normality Test*/
skewness=sumcubed/((T-1)*(variance^(3/2)));
kurtosis=sumquad/((T-1)*(variance^2));
JB=((T-1)/6)*(skewness^2+((kurtosis-3)^2)/4);

if JB < 5.99;
normality = "pass";
else;
normality = "fail";
endif;

/*Log Results - Normality Test*/
log_skewness=logsumcubed/((T-1)*log_variance^(3/2));
log_kurtosis=logsumquad/((T-1)*log_variance^2);
log_JB=(T-1)*(log_skewness^2+((log_kurtosis-3)^2)/4)/6;

if log_JB < 5.99;
log_normality = "pass";
else;
log_normality = "fail";
endif;

/*Random Walk 1:*/
/*Creating a 1247x11 matrix of returns rt, rt-1,..., rt-10*/
r_matrix=zeros(T-(lags+1),(lags+1));
m=1;
do until m>(lags+1);
      r_matrix[.,m]=log_ret[(12-m):(T-m),1];
      m=m+1;
endo;
cov_matrix=vcx(r_matrix);

/*Making a 10x1 matrix of Pearson Coefficient Values:*/
p_matrix=zeros(lags,1);
z_matrix=zeros(lags,1);
f=1;
do until f>lags;
      p_matrix[f,1]=cov_matrix[1,(f+1)]/(cov_matrix[1,1]^(1/2)*cov_matrix[(f+1),(f+1
)]^(1/2));
      z_matrix[f,1]=p_matrix[f,1]/((1/(T-11))^(1/2));
      f=f+1;
endo;

/*Box-Pierce Q-statistic*/
Qm=zeros(lags,1);
Qm_counter=1;
do until Qm_counter>lags;
      i=1;
      do until i>Qm_counter;
            Qm[Qm_counter,1]=Qm[Qm_counter,1]+((T-11)*(p_matrix[i,1]^2));
            i=i+1;
      endo;
      Qm_counter=Qm_counter+1;
endo;

/*RW3 Testing*/
VR=zeros((lags),1)+1;
```

```
q=1;
do until q>(lags);
      k=1;
      do until k>(q-1);
            VR[q,1]=VR[q,1]+2*(1-(k/(q-1)))*p_matrix[k,1];
            k=k+1;
      endo;
      q=q+1;
endo;

/* SD for VR... NOTE: k = q-1*/
gamma_tot = zeros((lags-1),1);
gamma_top = zeros((lags-1),1);
gamma_bottom = zeros((lags-1),1);
log_mean2=meanc(log_ret[1:(T-1),1]);

k=1;
do until k>(lags-1);
      j=(k+1);
      do until j>(T-1);
            gamma_top[k,1]=gamma_top[k,1]+(T-1)*((log_ret[j,1]-
log_mean2)^2*(log_ret[(j-k),1]-log_mean2)^2);
            j=j+1;
      endo;
      j=1;
      do until j>(T-1);
            gamma_bottom[k,1]=gamma_bottom[k,1]+(log_ret[j,1]-log_mean2)^2;
            j=j+1;
      endo;
      k=k+1;

endo;

gamma_tot=gamma_top./(gamma_bottom^2);
theta=zeros((lags),1);
q=1;
do until q>(lags);
      k=1;
      do until k>(q-1);
            theta[q,1]=theta[q,1]+gamma_tot[k,1]*(2*(1-(k/q)))^2;
            k=k+1;
      endo;
      q=q+1;
endo;

standardized=((T-1)^(1/2))*(VR[3:lags,1]-1)./(theta[2:(lags-1)]^(1/2));

RW3_test=zeros((lags-2),2);
i=1;
do until i>(lags-2);
      if (standardized[i,1]^2)^(1/2)>1.96;
            RW3_test[i,1]="Reject";
        RW3_test[i,2]=1;
      else;
            RW3_test[i,1]="RW3";
        RW3_test[i,2]=0;
      endif;
i=i+1;
endo;
```

```
/*Forecast Predictability based on VR result*/
i=1;
predictability=0;
do until i>(lags-2);
    if RW3_test[i,2]>0;
        predictability=i;
    endif;
i=i+1;
endo;

/*Assign returns based on predictability*/

/*filling in the matrix of returns for daily, weekly and monthly data*/
rt_plus1=zeros((T-lags-1),1);
i=1;
do until i>(T-lags-1);
      rt_plus1[i,1]=meanc(log_ret[i:(i+lags),1]);
      i=i+1;
endo;

rt=zeros((T-lags-1),lags);
if predictability>0;
    i=1;
    do until i>lags;
       rt[.,i]=log_ret[i:(T-1-(lags+1)+i),1];
       i=i+1;
    endo;

    reg_x=zeros((T-lags-1),predictability);
    reg_y=rt[.,2];
    i=1;
    do until i>(predictability);
        reg_x[.,i]=rt[.,(i+2)];
        i=i+1;
    endo;

    { vnam, m, b, stb,
      vc, stderr, sigma,
      cx, rsq, resid, dwstat } = ols(0,reg_y,reg_x);
    i=1;
    rt[.,1]=zeros(T-lags-1,1);
    rt[.,1]=rt[.,1]+b[1,1];

    do until i>predictability;
          rt[.,1]=rt[.,1]+rt[.,(i+1)].*b[(i+1),1];
          i=i+1;
    endo;
endif;

rt_plus1[.,1]=exp(rt_plus1)-1;
mean1=meanc(rt_plus1[.,1]);

if predictability>0;
    rt_plus1[.,1]=exp(rt[.,1])-1;
    rt1=meanc(rt[.,1]);
    rt1=exp(rt1)-1;
else;
    rt1=mean1;
endif;
```

```
if T<100;
    mor_m[.,monthly_i]=rt_plus1[.,1];
elseif T>400;
    mor_d[.,daily_i]=rt_plus1[.,1];
else;
    mor_w[.,weekly_i]=rt_plus1[.,1];
endif;

/*To get result with RW assumption, 5th column in should be changed from "rt1" to
"mean1"*/

if T<100;
monthly_matrix[monthly_i,1]=file_count;
monthly_matrix[monthly_i,2]=file_name;
monthly_matrix[monthly_i,3]="Monthly";
monthly_matrix[monthly_i,4]=3;
monthly_matrix[monthly_i,5]=rt1;
monthly_matrix[monthly_i,6]=predictability;
monthly_matrix[monthly_i,7]=mean1;
monthly_i=monthly_i+1;
elseif T>499;
daily_matrix[daily_i,1]=file_count;
daily_matrix[daily_i,2]=file_name;
daily_matrix[daily_i,3]="Daily";
daily_matrix[daily_i,4]=1;
daily_matrix[daily_i,5]=rt1;
daily_matrix[daily_i,6]=predictability;
daily_matrix[daily_i,7]=mean1;
daily_i=daily_i+1;
else;
weekly_matrix[weekly_i,1]=file_count;
weekly_matrix[weekly_i,2]=file_name;
weekly_matrix[weekly_i,3]="Weekly";
weekly_matrix[weekly_i,4]=2;
weekly_matrix[weekly_i,5]=rt1;
weekly_matrix[weekly_i,6]=predictability;
weekly_matrix[weekly_i,7]=mean1;
weekly_i=weekly_i+1;
endif;

file_count=file_count+1;
endo;
/*End of single file parsing*/

/*Start of portfolio building*/
oldfmt = formatnv("*.*lf" ~ 8 ~ 4);
let mask[1,7]= 1 0 0 1 1 1 1;
d=printfmt(daily_matrix,mask);
print;
d=printfmt(weekly_matrix,mask);
print;
d=printfmt(monthly_matrix,mask);
print;
call formatnv(oldfmt);

/*Reminder: mor_w, mor_m, mor_d for Matrix of net Returns - with different
frequencies*/
/*Risk Free Returns: Daily, Weekly, Monthly*/
rf_d=1.017^(1/260)-1;
```

```
rf_w=1.017^(1/52)-1;
rf_m=1.017^(1/12)-1;


/*-------------------------------------------------------------*/
/*                         Daily:                              */
/*-------------------------------------------------------------*/
Y=(1.2);
X=(1);
daily_Y=zeros(15,4);
g=1;
do until g>15;

/*Inverse of the Variance/covariance Matrix*/
v_ij=vcx(mor_d);
v_ij=inv(v_ij);

Wpi=zeros((daily_i-1),1);
a=zeros((daily_i-1),1);
b=zeros((daily_i-1),1);
c=zeros((daily_i-1),1);
/*Weight of stock i in portfolio p Wpi*/
i=1;
do until i>(daily_i-1);
    j=1;
    do until j>(daily_i-1);
        Wpi[i,1]=Wpi[i,1]+ v_ij[i,j]*(daily_matrix[j,5]-rf_d);
        a[i,1]=a[i,1]+v_ij[i,j]*(daily_matrix[j,5]);
        b[i,1]=b[i,1]+v_ij[i,j]*(daily_matrix[j,5])*(daily_matrix[i,5]);
        c[i,1]=c[i,1]+v_ij[i,j];
        j=j+1;
    endo;
    i=i+1;
endo;
i=1;
positive_sum=0;
negative_sum=0;
do until i>rows(Wpi);
    if Wpi[i,1]>0;
        positive_sum=positive_sum+Wpi[i,1];
    else;
        negative_sum=negative_sum+Wpi[i,1];
    endif;
    i=i+1;
endo;
a=sumc(a);
b=sumc(b);
c=sumc(c);
D=(c*rf_d^2-2*a*rf_d+b);
E= (rf_d+(Y/X)*D-D)/(1+D);
theta_k=2*(Y-X*(1+E))/X;

/*Solving for Weight of Risk Free Asset, E[Zp*], Var[P*] and RRA associated with
them*/
Wp0=1-(theta_k/2)*(a-c*rf_d);
epr=rf_d+(theta_k/2)*D;
varp=(theta_k/2)^2*D;
RRA=(X*(1+epr))/(Y-X*(1+epr));

print "--------------------------------------------------------------------------------";
```

```
print "Daily:";
print "+ve" positive_sum*(theta_k/2);
print "-ve" negative_sum*(theta_k/2);
print "Risky Asset Weights: " Wpi'.*(theta_k/2);
print;
print "Risky Sum: " sumc(Wpi.*(theta_k/2));
print "Risk Free: " Wp0;
print;
print "a b c: " a b c;
print "D E" D E;
print "Theta:     " theta_k;
print "E[Zp]=   " epr;
print "Var[Zp]= " varp;
print "RRA =    " RRA;
print;

daily_Y[g,1]=epr;
daily_Y[g,2]=varp;
daily_Y[g,3]=RRA;
daily_Y[g,4]=Y;
Y=Y+0.1;
g=g+1;
endo;

print "     E[Zp]      Var[Zp]      RRA       Y";
print daily_Y;

/*--------------------------------------------------------------*/
/*                         Weekly:                              */
/*--------------------------------------------------------------*/
Y=(1.5);
X=(1);
weekly_Y=zeros(15,4);
g=1;
do until g>15;

v_ij=vcx(mor_w);
v_ij=inv(v_ij);
Wpi=zeros((weekly_i-1),1);
a=zeros((weekly_i-1),1);
b=zeros((weekly_i-1),1);
c=zeros((weekly_i-1),1);
/*Weight of stock i in portfolio p Wpi*/
i=1;
do until i>(weekly_i-1);
    j=1;
    do until j>(weekly_i-1);
        Wpi[i,1]=Wpi[i,1]+ v_ij[i,j]*(weekly_matrix[j,5]-rf_w);
        a[i,1]=a[i,1]+v_ij[i,j]*(weekly_matrix[j,5]);
        b[i,1]=b[i,1]+v_ij[i,j]*(weekly_matrix[j,5])*(weekly_matrix[i,5]);
        c[i,1]=c[i,1]+v_ij[i,j];
        j=j+1;
    endo;
    i=i+1;
endo;

a=sumc(a);
b=sumc(b);
c=sumc(c);
rra_graph=zeros(15,3);
```

```
/*Solving for E_m = E[Zp] and then sub it in to solve for Theta  */
D=(c*rf_w^2-2*a*rf_w+b);
E= (rf_w+(Y/X)*D-D)/(1+D);
theta_k=2*(Y-X*(1+E))/X;

/*Solving for Weight of Risk Free Asset, E[Zp*], Var[P*] and RRA associated with
them*/
Wp0=1-(theta_k/2)*(a-c*rf_w);
epr=rf_w+(theta_k/2)*D;
varp=(theta_k/2)^2*D;
RRA=(X*(1+epr))/(Y-X*(1+epr));

print "--------------------------------------------------------------------------------";
print "Weekly:";
print "Risky Asset Weights: " Wpi'.*(theta_k/2);
print;
print "Risky Sum: " sumc(Wpi.*(theta_k/2));
print "Risk Free: " Wp0;
print;
print "a b c: " a b c;
print "D E" D E;
print "Theta:     " theta_k;
print "E[Zp]=   " epr;
print "Var[Zp]= " varp;
print "RRA =    " RRA;
print;

weekly_Y[g,1]=epr;
weekly_Y[g,2]=varp;
weekly_Y[g,3]=RRA;
weekly_Y[g,4]=Y;
Y=Y+0.2;
g=g+1;
endo;

print "     E[Zp]      Var[Zp]      RRA       Y";
print weekly_Y;

/*------------------------------------------------------------*/
/*                        Monthly:                            */
/*------------------------------------------------------------*/

Y=(1.5);
X=(1);
monthly_Y=zeros(15,4);
g=1;
do until g>15;

v_ij=vcx(mor_m);
v_ij=inv(v_ij);
Wpi=zeros((monthly_i-1),1);
a=zeros((monthly_i-1),1);
b=zeros((monthly_i-1),1);
c=zeros((monthly_i-1),1);

/*Weight of stock i in portfolio p Wpi*/
i=1;
do until i>(monthly_i-1);
    j=1;
```

```
    do until j>(monthly_i-1);
        Wpi[i,1]=Wpi[i,1]+ v_ij[i,j]*(monthly_matrix[j,5]-rf_m);
        a[i,1]=a[i,1]+v_ij[i,j]*(monthly_matrix[j,5]);
        b[i,1]=b[i,1]+v_ij[i,j]*(monthly_matrix[j,5])*(monthly_matrix[i,5]);
        c[i,1]=c[i,1]+v_ij[i,j];
        j=j+1;
    endo;
    i=i+1;
endo;

a=sumc(a);
b=sumc(b);
c=sumc(c);
rra_graph=zeros(15,3);

/*Solving for E_m = E[Zp] and then sub it in to solve for Theta  */
D=(c*rf_m^2-2*a*rf_m+b);
E= (rf_m+(Y/X)*D-D)/(1+D);
theta_k=2*(Y-X*(1+E))/X;

/*Solving for Weight of Risk Free Asset, E[Zp*], Var[P*] and RRA associated with
them*/
Wp0=1-(theta_k/2)*(a-c*rf_m);
epr=rf_m+(theta_k/2)*D;
varp=(theta_k/2)^2*D;
RRA=(X*(1+epr))/(Y-X*(1+epr));

print "-------------------------------------------------------------------------";
print "Monthly:";
print "Risky Asset Weights: " Wpi'.*(theta_k/2);
print;
print "Risky Sum: " sumc(Wpi.*(theta_k/2));
print "Risk Free: " Wp0;
print;
print "a b c: " a b c;
print "D E" D E;
print "Theta:     " theta_k;
print "E[Zp]=   " epr;
print "Var[Zp]= " varp;
print "RRA =    " RRA;
print "-------------------------------------------------------------------------";
print;

monthly_Y[g,1]=epr;
monthly_Y[g,2]=varp;
monthly_Y[g,3]=RRA;
monthly_Y[g,4]=Y;
Y=Y+1;
g=g+1;
endo;

print "     E[Zp]      Var[Zp]      RRA       Y";
print monthly_Y;
```