Tutorial 1

Question 1

(i) Equation 1 represents the budget constraint

(ii) • x_a and x_b are variables

• P_a and P_b are parameters (constants), so is M

(iii) 1. We have the following:

$$P_a x_a + P_b x_b = M$$

$$P_b x_b = M - P_a x_a$$

$$x_b = \frac{M}{P_b} - \frac{P_a}{P_b} x_a \quad (1)$$

vertical intercept — slope

In (1), when $x_b = 0$

$$\frac{M}{P_b} - \frac{P_a}{P_b} x_a = 0$$

$$\frac{P_a}{P_b} x_a = \frac{M}{P_b}$$

$$x_a = \frac{M}{P_a}$$

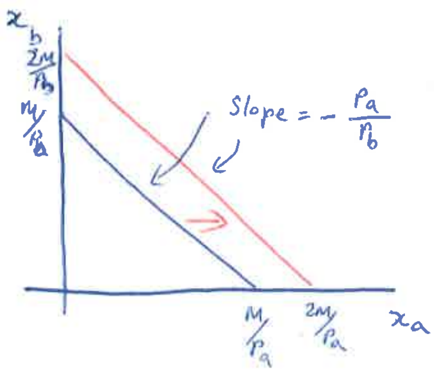
Horizontal intercept

2. If M doubles, we have the following:

$$P_a x_a + P_b x_b = 2M$$

$$\Rightarrow x_b = \frac{2M}{P_b} - \frac{P_a}{P_b} x_a \quad (2) \Rightarrow \text{outward shift}$$

Intercept has doubled — slope is unchanged



(iv) Opportunity cost of good a in terms of good b is $\frac{P_a}{P_b} = \frac{2}{6} = 1/3$.
In other words, you would need to give up 1/3 of good b to get 1 unit of good a.

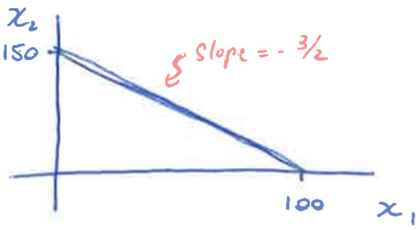
Question 2

(i) Choice set: $3x_1 + 2x_2 \leq 300$; Budget constraint: $3x_1 + 2x_2 = 300$

(ii) $3x_1 + 2x_2 = 300$

$2x_2 = 300 - 3x_1$

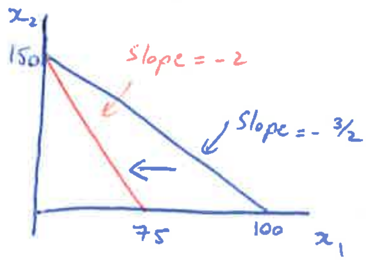
$x_2 = 150 - \frac{3}{2}x_1$ (1)



(iii) The slope of the budget line represents the opportunity cost of purchasing 1L of milk in terms of water, i.e. if you purchase 1 more litre of milk, you lose 1.5 litres of water.

(iv) A per unit tax t on good 1 yields the following:

$(P_1 + t)x_1 + P_2 x_2 = M \Rightarrow (3+1)x_1 + 2x_2 = 300$



$2x_2 = 300 - 4x_1$

$x_2 = 150 - 2x_1$

Slope is steeper

(v) After buying 5L of milk, price of milk changes from \$3 to \$1.50.

Cost of 5L of milk = $5 \times 3 = \$15$

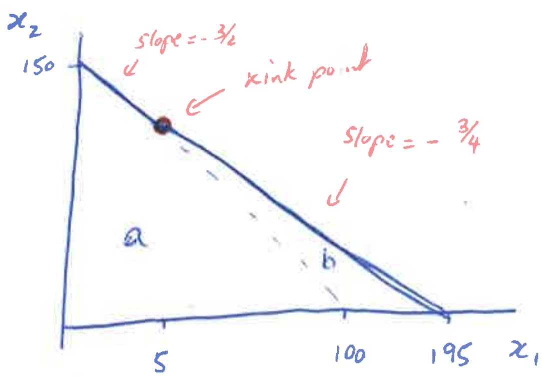
We know when $x_1 \leq 5$, we have $3x_1 + 2x_2 = 300$

When $x_1 > 5$, we have the following:

$1.5x_1 + 2x_2 = 300 - 15$

$1.5x_1 + 2x_2 = 285$, not concerned with vertical intercept

$\Rightarrow x_2 = 142.5 - 0.75x_1$, when $x_2 = 0 \Rightarrow x_1 = 190$, so total affordable $x_1 = 190 + 5 = 195$



choice set expands from a to a+b

TUTORIAL 2 - Solutions

Tastes

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Intermediate Microeconomics 23567 - Autumn 2017

Key Concepts

1. Monotonicity

⇒ (10,10) > (10,9)

slope of IC

2. Convexity

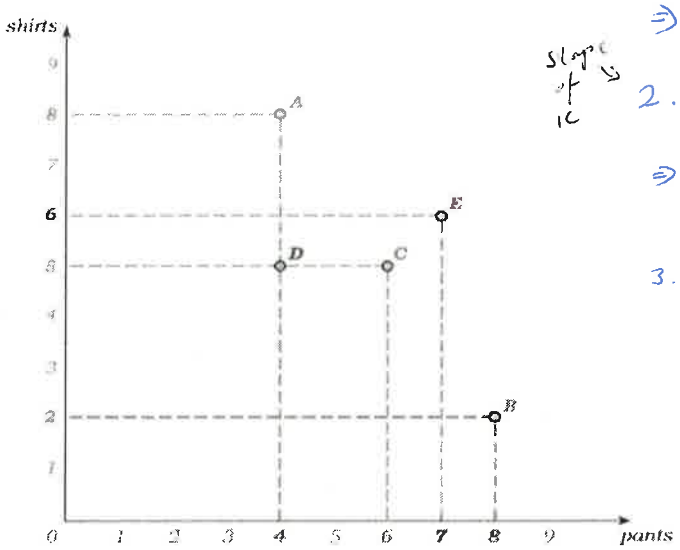
⇒ Average better than extremes

3. Transitivity

A > B & B > C ⇒ A > C

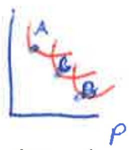
Question 1

Consider the following graph. Note that C is a weighted average of A and B.



a. Can you rank bundles A, B and C using only the *monotonicity* assumption?
Answer: NO
* They have differing levels of pants and shirts

b. Can you rank bundles A, B and C if you also assume that tastes satisfy *convexity*?
Answer: NO because nothing is said about the relation between A and B



c. Combining the convexity and monotonicity assumptions, can you now conclude something about the relationship between the pairs E and A and E and B?

No! ⇒ **Answer:** We can only apply the convexity assumption if we know some pair of bundles we are indifferent between—because convexity says that, when faced with bundles we are indifferent between, we prefer averages of such bundles (or at the very least like averages just as much). So, without knowing more, I can't use monotonicity and convexity to say anything about how A and E (or B and E) are related to one another.

① Need to know what bundles the person is indifferent between

⇒ Not enough information

• Indifferent between A & B
• C is average of A & B ⇒ C ≿ A or B ⇒ E ≻ A & E ≻ B
• E has more of everything than C

d. If you know that I am indifferent between *A* and *B*, can you conclude something about the relationship between the pairs E and A, and E and B?
Answer: If we know that I am indifferent between A and B, on the other hand, then I know that C is at least as good as A and B because C is the average between A and B. Since E has more of everything than C, we also know from monotonicity that E is better than C. So E is better than C which is at least as good as A and B. By transitivity, that implies that E is better than A and B.

Only need ⇒ Monotonicity, not convexity

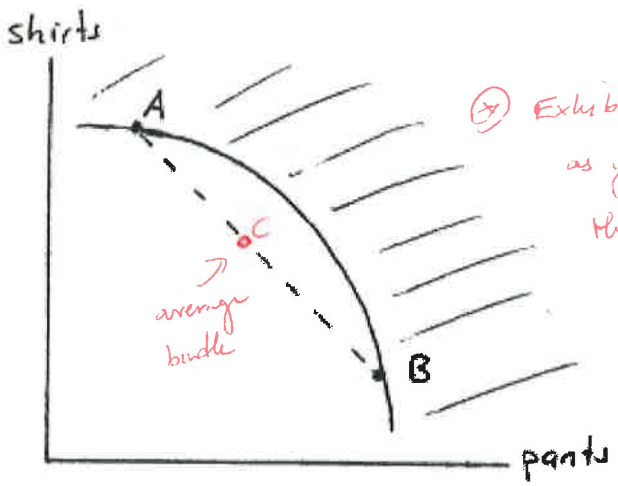
∴ A ~ B ⇒ B ≿ D because A ≿ D

e. Knowing that I am indifferent between A and B, can you now conclude something about how B and D are ranked by me? In order to reach this conclusion, do you necessarily have to invoke the convexity assumption?
Answer: By just invoking the monotonicity assumption, I know that A is at least as good as D since it has more of one good and the same of the other. If A is indifferent to B, I then also know (by transitivity) that B is at least as good as D. So, it is not necessary to invoke the convexity assumption. (even though in principle we could use it to get to the same conclusion). Note that invoking convexity won't actually allow me to say anything beyond what I can already conclude by invoking monotonicity. This is so because convexity only implies that C is at least as good as A and B (it is strict convexity that implies that C is definitely better than A and B).

Question 2 (*) optional

Suppose extremes are better than averages (while all the other standard assumption about tastes hold). What would an indifference curve look like? Would it still imply diminishing marginal rates of substitution?

Answer: The indifference curve would bend outward as in the graph below. Note it is still doward sloping and still the shaded area to the northeast of the indifference curve would contain all the better bundles (because of monotonicity). But the line connecting A and B — which contains averages between A and B — does not lie in this "better" region. Therefore, averages are worse than extremes.
The slope of this indifference curve is then shallow at A and becomes steeper as we move along the indifference curve to B. Thus, the marginal rate of substitution is no longer diminishing long the indifference curve — and the indifference curve exhibits increasing marginal rates of substitution.



(*) Exhibits increasing MRS, as you get more pants the slope becomes steeper

Question 3 – On the assumption of monotonicity , "Goods" and "Bads" .

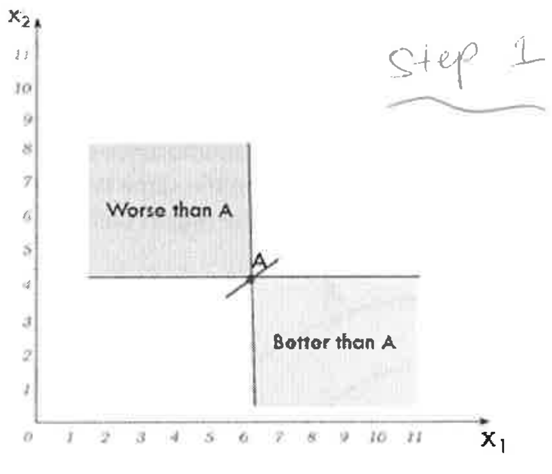
The assumption of monotonicity (i.e. "more is better or at least no worse") is what makes an item "a good" rather than "a bad". Indeed, if we assume that a consumer likes to have more of an item, that's because that item is good to him or her. But how do we deal with items that we do not like? In economics, items that a consumer does not like are called "bads". For a bad, the assumption of monotonicity is obviously violated. In fact, for a bad, the opposite holds: the less, the better.

So, suppose that to an individual having more of good 1 is good, while having more of good 2 is a bad. For example, this individual is a gardener, good 1 are flowers, and good 2 are termites. Suppose also that we know the tastes of this individual also satisfy strong convexity.

a. Which of the standard assumptions about tastes is violated?
   Answer: Monotonicity is violated

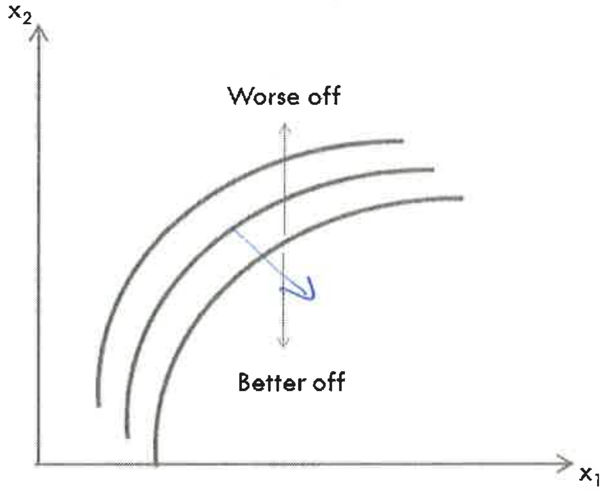
b. In a graph with good 1 on the horizontal axis and good 2 on the vertical, plot a map of indifference curves for this individual. With an arrow, show the direction where we find bundles that make this individual better off.

Answer: First, note that indifference curves must be upward or positively sloping. There are two alternative ways to show this. The first one is shown in the first graph below. Pick a generic bundle (say A) and then identify those bundles that are certainly better than A, and those other bundles that are certainly worse than A. Then you can conclude that those bundles that are as good as A must lie in the white regions. Thus the indifference curve through A must lie in that region too, which makes it positively sloped



If I also know that the tastes of this individual satisfy strong convexity, I know that he strictly prefers averages to the extremes. Thus his indifference curves **must** exhibit the shape of those in the graph below. Why? Because if you take any two bundles on the same indifference curve, and then take a average of the two, you see that the average is better (because it lies in the region where we have bundles that make the individual better off).

Step 2 : Factoring in Convexity



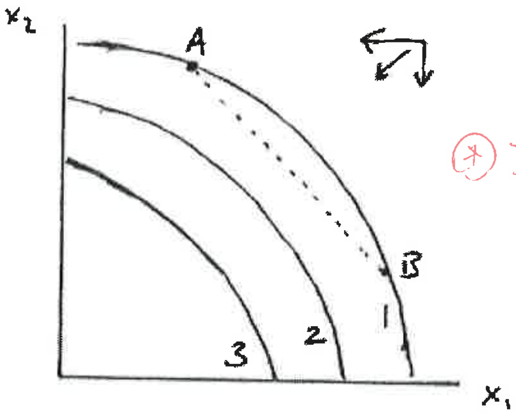
MRS = WTA of the "bad" for unit of the "good"

c. Is the MRS negative or positive? How would you define the MRS in this case?
Answer: In this case, the MRS would represent the units of good 2 (the "bad") that you are willing to **accept** in order to get an additional unit of the good 1 (the "good").

Question 4 – bundles of "Bads" (*) optional

Suppose you do not like cigarettes and whisky. That is, to you, less is better than more, both for cigarettes and whisky. Suppose also that, to you, averages are strictly better than extremes. Draw three indifference curves (with numerical labels) that would be consistent with your tastes.

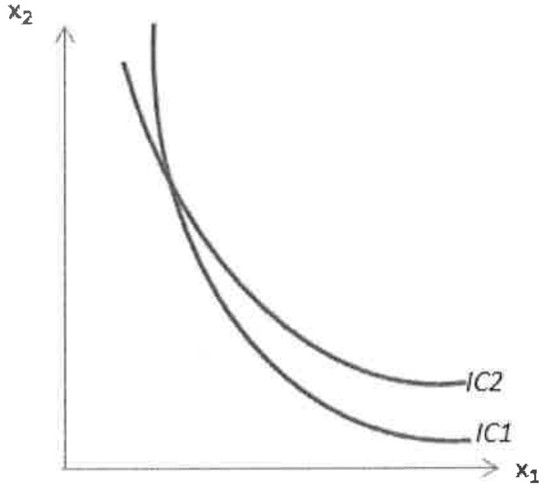
Answer: The graph below illustrates three such curves. First, note that since less is better (and this applies to both cigarettes and whisky), the consumer becomes better off in the direction of the arrows at the top right of the graph. Consistently, the numbers accompanying the indifference curves must be increasing as we approach the origin. Second, note that the shape of the indifference curves is consistent with averages being better than extremes. Indeed, if I take A and B that lie on the same indifference curve, the line connecting them (which contains averages of the two) lies fully in the region that is more preferred.



(*) Draw Graph!

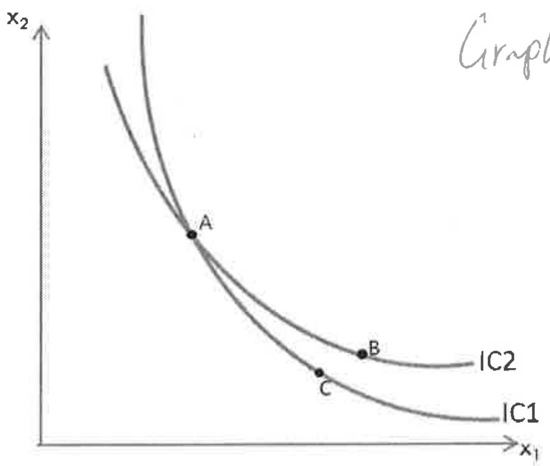
Question 6

Consider the following graph:



Suppose that the two indifference curves IC1 and IC2 are representing the tastes of the same individual over bundles of good1 and good 2. Show that in this case the assumption of transitivity is necessarily violated. Note that proving this result is equivalent to prove the statement in slide 30 of lecture 2 that if the assumption of transitivity holds, then the ICs of the same individual cannot intersect.

Answer: Consider the following graph:



Graph this

A is as good as B because they lie on the same indifference curve. A is as good as C because they lie on the same indifference curve. If transitivity held, then the consumer would be indifferent between B and C. However, since B and C are on two different indifferent curves, this consumer is not indifferent between B and C. Therefore transitivity is violated.

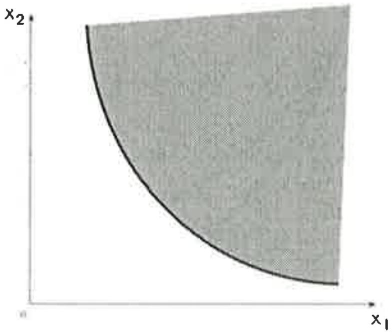
1. $A \sim B$
2. $A \sim C$ $\Rightarrow B \sim C$ by transitivity, however, this is not true as B & C are on 2 different IC's!

∴ Transitivity does not hold!

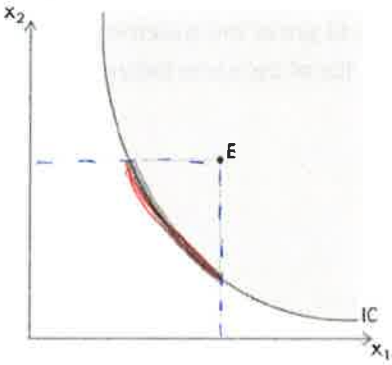
1. Monotonicity
2. Convexity
3. Transitivity
4. Completeness
5. Continuity

Question 5

Assume that the 5 assumptions about tastes hold. Now, consider the graph below showing an indifference curve. Show that all bundles that lie to the north-east of the indifference curve (i.e. in the shaded area) are strictly preferred to ***all*** bundles that lie on the indifference curve. Note that showing this result proves that a consumer for which the assumption of monotonicity holds is better off as he moves towards indifference curves that lie in the north-east region of our graph (slide 26 of lecture 2).



Answer: Pick any point that lies in the shaded area, say E.



• E ≻ [Highlighted part]
• [Highlighted part] ~ IC
⇒ E ≻ IC ! by transitivity

By monotonicity, E is strictly better than any bundle on the highlighted part of the indifference curve.

But any bundle on the highlighted part of the indifference curve is as good as any bundle on the non-highlighted part of the indifference curve.

Then, by transitivity, E is strictly better than any bundle on the non-highlighted part of the indifference curve.

We have thus shown that E is strictly preferred to all bundles on the highlighted and non-highlighted part of the indifference curve. Thus E is strictly better than all bundles that lie on the indifference curve.

Note you can apply the same logic to any bundle that lies in the shaded area above the indifference. Thus you can conclude that all bundles that lie in the shaded area above the indifference curve are strictly preferred to all bundles that lie on the indifference curve.

Tutorial 3

Question 1

We have $u(x_1, x_2) = 3x_1^{2/3} x_2^{1/3}$

a) $u_A(1, 27) = 3(1)^{2/3}(27)^{1/3} = 9$; $u_B(27, 1) = 3(27)^{2/3}(1)^{1/3} = 27$

$\therefore u_B > u_A$!

b) when $3x_1^{2/3} x_2^{1/3} = 9$

$$x_1^{2/3} x_2^{1/3} = 3$$

$$x_2^{1/3} = 3/x_1^{2/3}$$

$$x_2 = 27/x_1^2$$

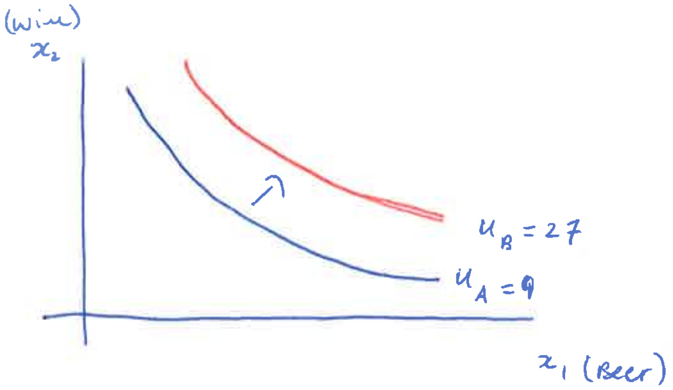
c) when $3x_1^{2/3} x_2^{1/3} = 27$

$$x_1^{2/3} x_2^{1/3} = 9$$

$$x_2^{1/3} = 9/x_1^{2/3}$$

$$x_2 = \frac{729}{x_1^2}$$

d)



e) $u(x_1, x_2) = 3x_1^{2/3} x_2^{1/3}$

$$du/dx_1 = 2x_1^{-1/3} x_2^{1/3} \quad ; \quad du/dx_2 = x_1^{2/3} x_2^{-2/3}$$

(*) $MRS = -\dfrac{du/dx_1}{du/dx_2} = -\dfrac{2x_1^{-1/3} x_2^{1/3}}{x_1^{2/3} x_2^{-2/3}} = -\dfrac{2x_2}{x_1}$

For $A = (1, 27) \Rightarrow MRS = -\frac{2 \times 27}{1} = -54$

For $B = (27, 1) \Rightarrow MRS = -\frac{2 \times 1}{27} = -2/27$

f) Recall, $MRS = -\dfrac{2x_2}{x_1}$. As x_1 increases, MRS decreases. Hence, Mark's tastes demonstrate diminishing MRS. As he gets more beer, he is less willing to give up wine.

Question 2

Now we have $u(x_1, x_2) = x_1 + x_2$

a) $u_A(1,27) = 1+27 = 28$; $u_B(27,1) = 27+1 = 28$

$\therefore u_A = u_B$!

b) & c) $u(x_1, x_2) = x_1 + x_2$

when $u = 28 \Rightarrow x_1 + x_2 = 28 \Rightarrow x_2 = 28 - x_1$

d)

e) $du/dx_1 = 1$; $du/dx_2 = 1$

$$MRS = -\frac{du/dx_1}{du/dx_2} = -\frac{1}{1} = -1$$

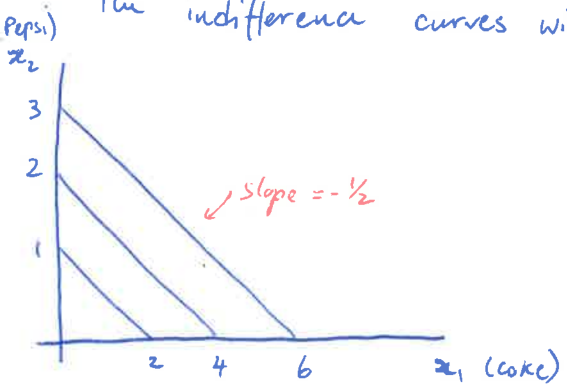
$\therefore$ John is willing to give up 1L of wine for 1L of beer.

f) John's MRS = -1 which is constant!

Question 3 [optional] * Skip!

We have that 2 coke = 1 pepsi

a) Coke & Pepsi are perfect substitutes, because they taste exactly the same. The indifference curves will be linear.



b) The MRS = -½, to get 1 more can of Coke you have to give up ½ a can of Pepsi.

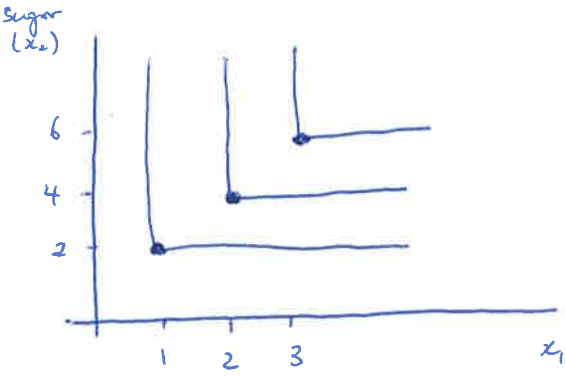
c) $u(x_1, x_2) = x_1 + 2x_2 \Rightarrow 2x_2 = u - x_1$

$\Rightarrow x_2 = \frac{u}{2} - \frac{x_1}{2}$

Question 4

we need 2 packs of sugar for each glass of tea.

a) Sugar & Tea are perfect complements:



$x_2 = 2x_1$

when $x_1 = 1$, $x_2 = 2$

$x_1 = 2$, $x_2 = 4$

$x_1 = 3$, $x_2 = 6$

b) Consumer is not willing to substitute sugar for Tea, as they are perfect complements.

c) suppose $u(x_1, x_2) = \min\{2x_1, x_2\}$

suppose you have 1 cup of Tea & 10 sugars:

$u(1, 10) = \min\{2 \times 1, 10\} = 2$, the utility you derive from Tea & Sugar is equal to the minimum number of good cups of tea you can make.

Question 5

a) we have $u(x_1, x_2) = x_1^2 x_2^2$ & $v(x_1, x_2) = x_1^{1/2} x_2^{1/2}$

$\partial u / \partial x_1 = 2x_1 x_2^2$; $\partial u / \partial x_2 = 2x_1^2 x_2$

$\partial v / \partial x_1 = \frac{1}{2} x_1^{-1/2} x_2^{1/2}$; $\partial v / \partial x_2 = \frac{1}{2} x_1^{1/2} x_2^{-1/2}$

$MRS_u = -\dfrac{2x_1 x_2^2}{2x_1^2 x_2} = -\dfrac{x_2}{x_1}$

$MRS_v = -\dfrac{\frac{1}{2} x_1^{-1/2} x_2^{1/2}}{\frac{1}{2} x_1^{1/2} x_2^{-1/2}} = -\dfrac{x_2}{x_1}$

$\therefore$ $MRS_u = MRS_v$, so tastes are the same!

b) we have $u(x_1, x_2) = x_1^{2/3} x_2^{1/3}$; $V(x_1, x_2) = x_1^{1/2} x_2^{1/2}$

$\partial u / \partial x_1 = \frac{2}{3} x_1^{-1/3} x_2^{1/3}$; $\partial u / \partial x_2 = \frac{1}{3} x_1^{2/3} x_2^{-2/3}$

$\partial v / \partial x_1 = \frac{1}{2} x_1^{-1/2} x_2^{1/2}$; $\partial v / \partial x_2 = \frac{1}{2} x_1^{1/2} x_2^{-1/2}$

$MRS_u = -\dfrac{\frac{2}{3} x_1^{-1/3} x_2^{1/3}}{\frac{1}{3} x_1^{2/3} x_2^{-2/3}} = -\dfrac{2x_2}{x_1}$

$MRS_v = -\dfrac{\frac{1}{2} x_1^{-1/2} x_2^{1/2}}{\frac{1}{2} x_1^{1/2} x_2^{-1/2}} = -\dfrac{x_2}{x_1}$

$\therefore$ $MRS_u \neq MRS_v \Rightarrow$ tastes are different!

Tutorial 4

Question 1

a) We have $u(x_1, x_2) = x_1^{1/3} x_2^{2/3}$ and $P_1 x_1 + P_2 x_2 = I$

$$\frac{du}{dx_1} = \frac{1}{3} x_1^{-2/3} x_2^{2/3} \; ; \; \frac{du}{dx_2} = \frac{2}{3} x_1^{1/3} x_2^{-1/3} \Rightarrow MRS = -\frac{\frac{1}{3} x_1^{-2/3} x_2^{2/3}}{\frac{2}{3} x_1^{1/3} x_2^{-1/3}} = -\frac{x_2}{2x_1}$$

Optimal bundle is chosen when, $MRS = P_1/P_2$ (1)

$P_1 x_1 + P_2 x_2 = I$ (2)

(*) Note $MRS = -\dfrac{du/dx_1}{du/dx_2}$

In (1) we have, $\dfrac{x_2}{2x_1} = P_1/P_2 \Rightarrow x_2 = \dfrac{2P_1 x_1}{P_2}$ (1')

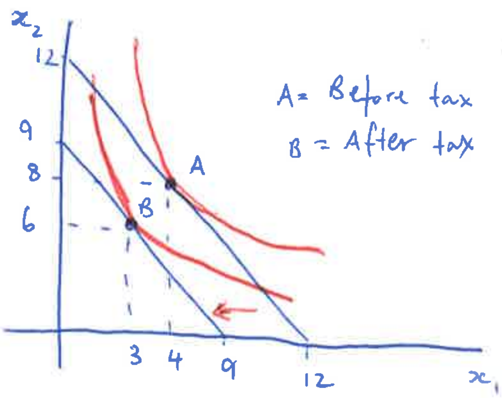
Substitute (1') into (2):

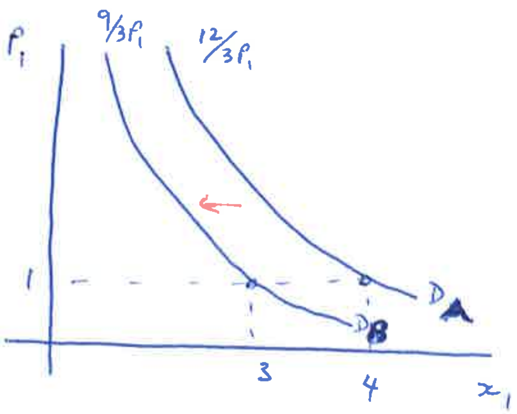
$$P_1 x_1 + P_2\left(\frac{2P_1 x_1}{P_2}\right) = I \Rightarrow 3P_1 x_1 = I \Rightarrow \boxed{x_1^* = \frac{I}{3P_1} \; ; \; x_2^* = \frac{2I}{3P_2}}$$

b) Before tax : $x_1 = \dfrac{I}{3P_1} = \dfrac{12}{3 \cdot 1} = 4$ & $x_2 = \dfrac{2I}{3P_2} = \dfrac{2 \cdot 12}{3 \cdot 1} = 8$

After tax : $x_1 = \dfrac{I}{3P_1} = \dfrac{12(1-0.25)}{3 \cdot 1} = 3$ & $x_2 = \dfrac{2I}{3P_2} = \dfrac{2 \cdot 12(1-0.25)}{3 \cdot 1} = 6$

c)



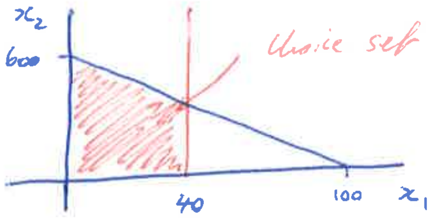


Question 2

a) For both families we have the following:

$$6x_1 + x_2 \leq 600 \quad \& \quad x_1 \leq 40 \ (2)$$

$$\Rightarrow x_2 \leq 600 - 6x_1 \ (1)$$



b) Let's assume $x_1 \leq 40$ is not binding.

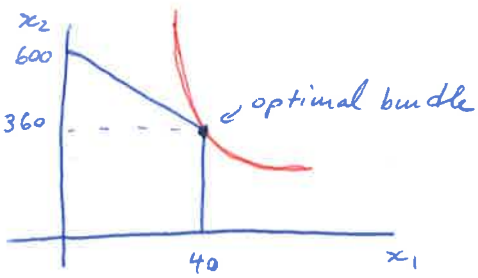
For family A we have the following:

$$\max_{\{x_1,x_2\}} x_1 x_2 \quad \text{s.t.} \quad x_2 = 600 - 6x_1 \Rightarrow \max_{\{x_1\}} x_1(600-6x_1) \Rightarrow \max_{\{x_1\}} 600x_1 - 6x_1^2$$

F.O.C. x_1

$$600 - 12x_1 = 0 \Rightarrow x_1^* = 50, \text{ however, } x_1 \leq 40 \therefore \text{ constraint on } x_1 \text{ is binding!}$$

$$\Rightarrow x_1^* = 40, \quad x_2^* = 600 - 6 \cdot 40 = 360$$



c) Let's assume $x_1 \leq 40$ is not binding!

For family B we have the following:

$$\max_{\{x_1,x_2\}} x_1^{1/4} x_2^{3/4} \quad \text{s.t.} \quad x_2 = 600 - 6x_1 \Rightarrow \max_{\{x_1\}} x_1^{1/4}(600-6x_1)^{3/4}$$

F.O.C. x_1

$$\tfrac{1}{4} x_1^{-3/4}(600-6x_1)^{3/4} - \tfrac{18}{4} x_1^{1/4}(600-6x_1)^{-1/4} = 0$$

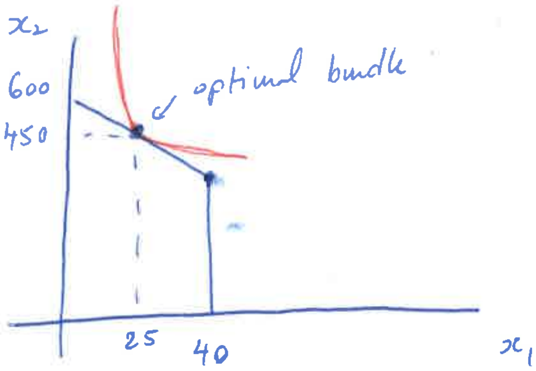
$$x_1^{-3/4}(600-6x_1)^{3/4} = 18 x_1^{1/4}(600-6x_1)^{-1/4}$$

$$18x_1 = 600 - 6x_1$$

$$24x_1 = 600$$

$$x_1^* = 25, \text{ satisfies } x_1 \leq 40!$$

$$x_2^* = 450$$



Question 2 : Alternative approach

c) Lets assume $x_1 \leq 40$ is not binding.

For family B we have the following:

$$\max_{\{x_1, x_2\}} \; x_1^{1/4} x_2^{3/4}$$

s.t.

$$6x_1 + x_2 = 600 \quad (1)$$

$$\partial u / \partial x_1 = 1/4 \, x_1^{-3/4} x_2^{3/4} \; ; \; \partial u / \partial x_2 = 3/4 \, x_1^{1/4} x_2^{-1/4} \Rightarrow MRS = -\frac{1/4 \, x_1^{-3/4} x_2^{3/4}}{3/4 \, x_1^{1/4} x_2^{-1/4}} = -\frac{x_2}{3x_1}$$

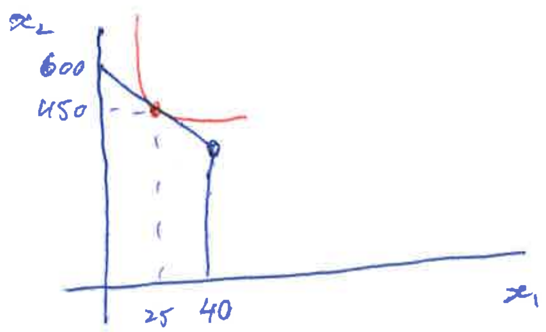
Family B achieve the optimal bundle when:

$$MRS = -\frac{P_1}{P_2} \Rightarrow -\frac{x_2}{3x_1} = -\frac{6}{1} \Rightarrow x_2 = 18x_1 \quad (2)$$

Substitute (2) into (1):

$$6x_1 + 18x_1 = 600 \Rightarrow 24x_1 = 600 \Rightarrow x_1^* = 25 \leq 40!$$

$$\Rightarrow x_2^* = 450$$



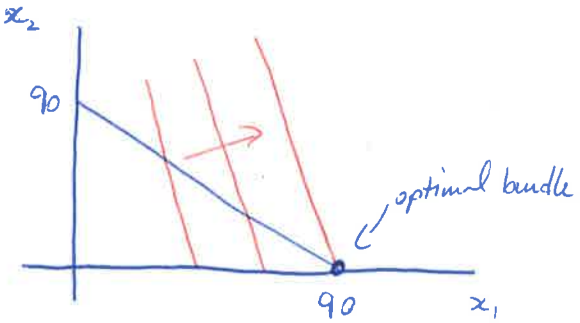
Question 3 (Perfect substitutes)

we have the following problem:

$$\max_{\{x_1, x_2\}} \quad 3x_1 + x_2 \quad \text{s.t.} \quad x_1 + x_2 = 90 \Rightarrow x_2 = 90 - x_1$$

$$\Rightarrow \max_{\{x_1\}} \quad 3x_1 + 90 - x_1 \Rightarrow \max_{\{x_1\}} \quad 2x_1 + 90$$

$\therefore \hat{x}_1 = 90, \ \hat{x}_2 = 0$



Question 4 (Perfect complements)

we have the following problem:

$$\max_{\{x_1, x_2\}} \quad \min(2x_1, x_2)$$

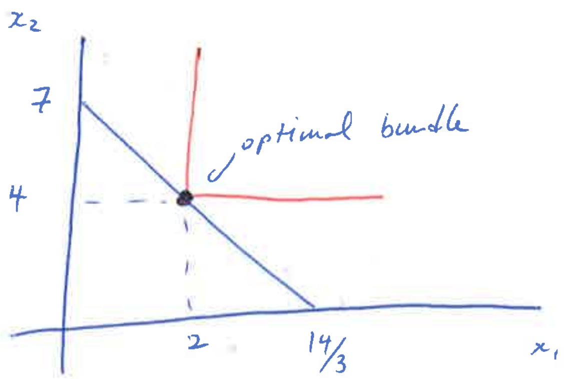
$$\text{s.t.}$$

$$3x_1 + 2x_2 = 14$$

Let $2x_1 = x_2$, $\therefore 3x_1 + 2 \cdot 2x_1 = 14$

$$7x_1 = 14$$

$$x_1^* = 2, \ x_2^* = 4$$



Question 3

a) we have the following:

$$\max_{\{x_1, x_2\}} x_1 x_2 \quad \Rightarrow \quad \max_{\{x_1\}} x_1\left(I/P_2 - P_1/P_2\, x_1\right)$$

s.t.

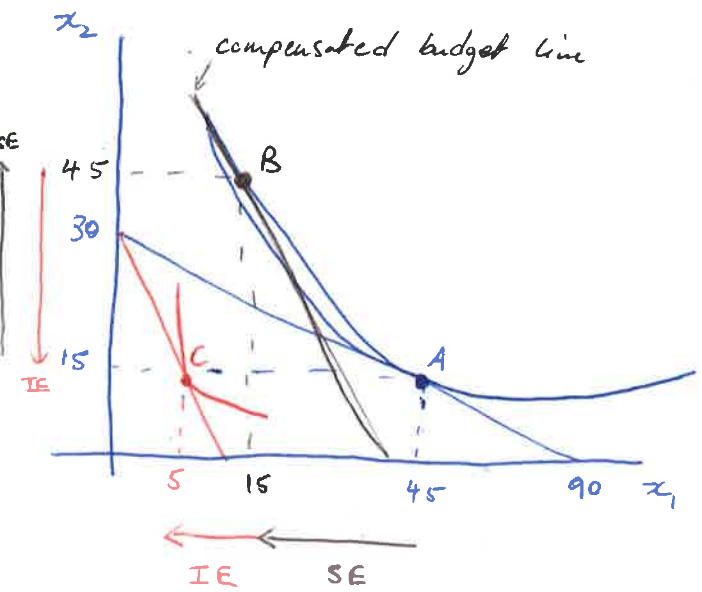
$$P_1 x_1 + P_2 x_2 = I \Rightarrow x_2 = \frac{I}{P_2} - P_1/P_2\, x_1$$

F.O.C. x_1

$$\frac{I}{P_2} - 2\frac{P_1}{P_2} x_1 = 0 \Rightarrow \overset{*}{x}_1 = \frac{I}{2P_1} \;\&\; \overset{*}{x}_2 = \frac{I}{2P_2} \text{ by symmetry}$$

b) when $P_1 = 1$, $P_2 = 3$ and $I = 90$;

$$\overset{*}{x}_1 = \frac{90}{2\cdot 1} = 45 \;\&\; \overset{*}{x}_2 = \frac{90}{2\cdot 3} = 15, \quad \therefore (\overset{*}{x}_1, \overset{*}{x}_2) = (45, 15)$$



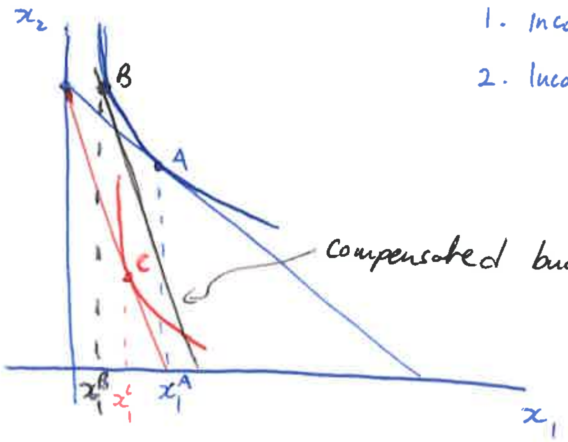
c) when $P_1 = 9$, $P_2 = 3$ and $I = 90$, $\overset{*}{x}_1 = 5$; $\overset{*}{x}_2 = 15$, $\therefore (\overset{*}{x}_1, \overset{*}{x}_2) = (5, 15)$

Tutorial 5

Question 1

a)

suppose that $\uparrow P_1$



For x_1 we know the following:

1. Income & substitution effects go in opposite directions
2. Income effect < substitution effect

SE : $A \rightarrow B$

IE : $B \rightarrow C$

TE : $A \rightarrow C$, so a reduction in demand!

$\therefore$ as P_1 increases, x_1 goes down, so law of demand is satisfied.

b)
- The good is regular because it satisfies the law of demand
- The good is inferior because the income & substitution effects go in opposite directions

$\therefore$ Good is called a regular inferior good

Question 2

True,

- If P_1 increases, purchasing power (real income) decreases, given x_2 is inferior, consumption of x_2 will go up with the reduced real income.
- If P1 increases, opportunity cost of x_2, i.e. P_2/P_1 decreases. Hence, substitution effect is positive and they will consume more x_2.

$\therefore$ No doubt that x_2 will increase as P_1 increases!

d) For compensated bundle, B, we know:

$$u(x_1^B, x_2^B) = u(A) \Rightarrow x_1 x_2 = 45 \cdot 15 = 675$$

$$\therefore \; x_1 x_2 = 675 \quad (1)$$

We must also have that at B the indifference curve is tangent to the compensated budget line:

$$MRS(x_1^B, x_2^B) = -\frac{p_1^{new}}{p_2} \Rightarrow -\frac{x_2}{x_1} = -9/3$$

$$\therefore \; \frac{x_2}{x_1} = 3 \quad (2) \Rightarrow x_2 = 3x_1$$

Substitute (2) into (1):

$$x_1 \cdot 3x_1 = 675 \Rightarrow x_1^2 = 225 \Rightarrow x_1^* = 15, \; x_2^* = 45$$

$$\therefore \; B = (15, 45)$$

For x_1: $SE = x_1^B - x_1^A = 15 - 45 = -30$

$IE = x_1^C - x_1^B = 5 - 15 = -10$

$TE = SE + IE = -30 - 10 = -40$

For x_2: $SE = x_2^B - x_2^A = 45 - 15 = 30$

$IE = x_2^C - x_2^B = 15 - 45 = -30$

$TE = SE + IE = 30 - 30 = 0$

e) x_1 is normal, SE & IE go in same direction! Alternatively look at Demand function

f) x_2 is normal, look at demand for x_2!

g) see graph

h) Recall, $A = (45, 15)$ & $C = (5, 15) \Rightarrow$ as P_1 increased x_2 stayed the same.
This is because the SE & IE perfectly counterbalanced eachother.

Tutorial 6

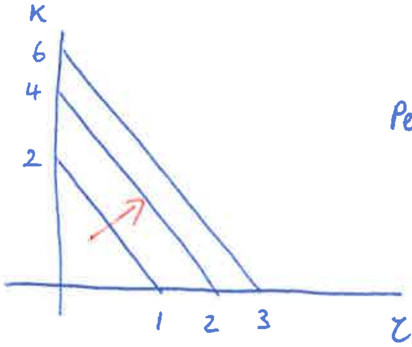
Question 1

a) We have $f(\ell, K) = 2\ell + K$

when $f(\ell, K) = 2 \Rightarrow 2\ell + K = 2$

$\Rightarrow K = 2 - 2\ell$

b)



Perfect substitutes

c) TRS is constant, i.e. the slope is constant. This is what makes ℓ & K perfect substitutes.

d) $f_\ell = 2$; $f_K = 1$

$TRS = -\frac{f_\ell}{f_K} = -\frac{2}{1} = -2$

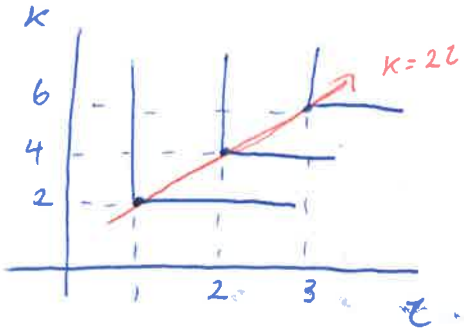
$\therefore$ TRS is constant!

Question 2

a) we have $f(\ell, K) = \min\{2\ell, K\}$

when $f(\ell, K) = 2 \Rightarrow \min\{2\ell, K\} = 2$

b) Firm will produce when $2\ell = K$



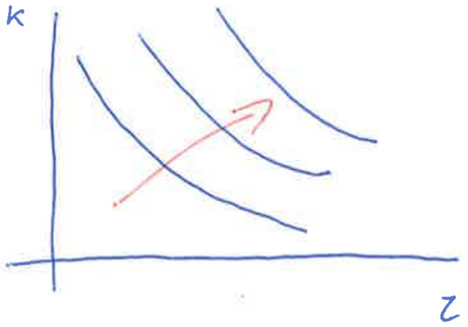
c) Here K & ℓ are perfect complements so the degree of substitutability is essentially zero, i.e. TRS = 0.

Question 3

a) we have $f(k,\ell) = \ell k$

when $f(k,\ell) = 2 \Rightarrow \ell k = 2 \Rightarrow k = \frac{2}{\ell}$

b)



c) TRS is diminishing with respect to ℓ, i.e. the slope changes as we move along the isoquant.

d) $f_\ell = k$; $f_k = \ell$

$TRS = -\frac{f_\ell}{f_k} = -\frac{k}{\ell}$

$\therefore$ TRS varies with respect to ℓ & k!

Question 4

a) we have $w = 1$, $r = 2$ & $f(\ell,k) = \ell k$

$f_\ell = k$; $f_k = \ell \Rightarrow TRS = -\frac{k}{\ell}$

Producer will produce when the following holds:

$$TRS = -\frac{w}{r} \Rightarrow -\frac{k}{\ell} = -\frac{1}{2}$$

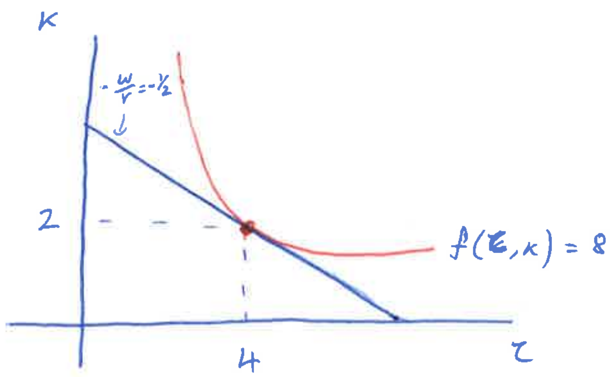
$$\Rightarrow k = \frac{\ell}{2} \quad (1)$$

Also $f(\ell,k) = 8 \Rightarrow \ell k = 8 \quad (2)$

substitute (1) into (2):

$$\frac{\ell^2}{2} = 8 \Rightarrow \ell^2 = 16$$

$$\ell^* = 4 ; k^* = 2$$

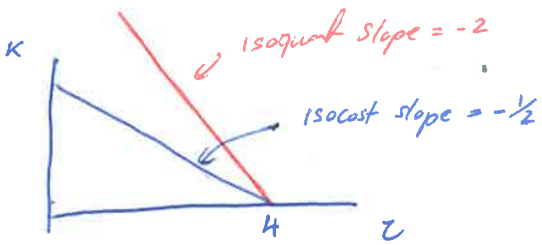


b) we have $w = 1$, $r = 2$ & $f(\ell,k) = 2\ell + k$.

Firms problem is the following:

$$\min_{\{\ell,k\}} \ell + 2k \quad \text{s.t.} \quad 2\ell + k = 8 \Rightarrow \ell = 4 - \frac{k}{2}$$

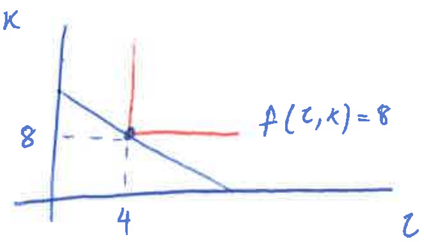
$$\Rightarrow \min_{\{k\}} 4 - \frac{k}{2} + 2k \Rightarrow \min_{\{k\}} 4 + \frac{3k}{2} \quad \therefore k^* = 0 ; \ell^* = 4$$



c) we have $w = 1$, $r = 2$ & $f(\ell,k) = \min\{2\ell, k\}$

Firm will set $2\ell = k$ (1)

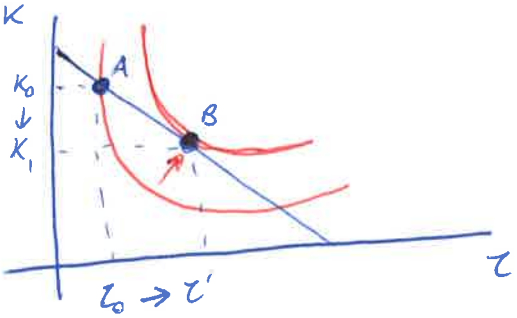
When $f(\ell,k) = 8 \Rightarrow k^* = 8$ & $\ell^* = 4$



Question 5

We know the $TRS = -3$ & $\frac{w}{r} = 2$

At the optimum $TRS = -\frac{w}{r}$, however, $-3 \neq -2$



we are currently at A but we want to get to B!

The firm should employ more labor & less capital!

Tutorial 7

Question 1

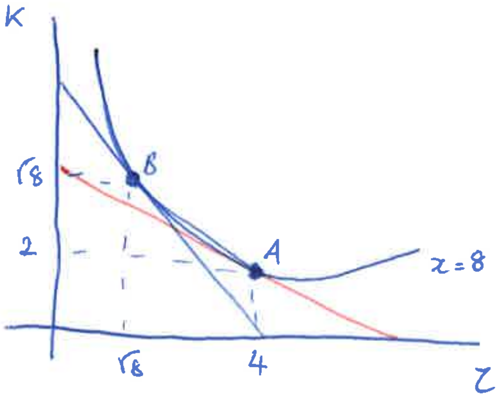
(i) We have the following:

$$\min_{\{k,\ell\}} k+\ell \quad \text{s.t.} \quad k\ell = 8 \Rightarrow k = 8/\ell \quad \Rightarrow \min_{\{\ell\}} 8/\ell + \ell \Rightarrow \min_{\{\ell\}} 8\ell^{-1}+\ell$$

F.O.C. ℓ

$$-8\ell^{-2}+1=0 \Rightarrow 8/\ell^2 = 1 \Rightarrow \ell^2 = 8 \Rightarrow \ell^* = \sqrt{8} \; ; \; k^* = \sqrt{8}$$

(ii)



(iii) Now we have the following:

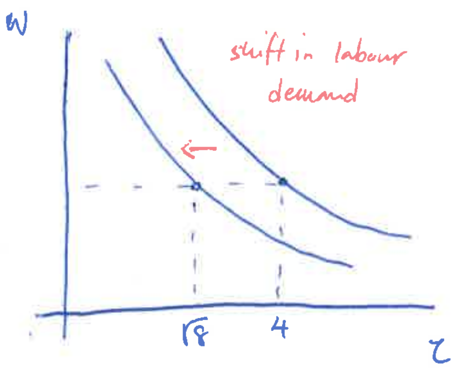
$$\min_{\{k,\ell\}} w\ell + rk \quad \text{s.t.} \quad k\ell = x \quad \Rightarrow \min_{\{\ell\}} w\ell + \frac{rx}{\ell}$$

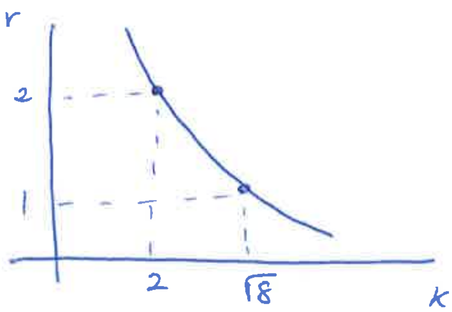
F.O.C. ℓ

$$w - \frac{rx}{\ell^2} = 0 \Rightarrow \ell^2 = \frac{rx}{w} \Rightarrow \ell^* = \sqrt{\frac{rx}{w}} \; ; \; k^* = \sqrt{\frac{wx}{r}} \text{ by symmetry}$$

(iv) when $w=1$, $r=2$ and $x=8$ we have the following:

$$\ell^* = \frac{4}{\sqrt{w}} \Rightarrow w = \frac{16}{\ell^2} \quad \& \quad k^* = \sqrt{\frac{8}{r}} \Rightarrow r = 8/k^2$$





Question 2

(i) When $w=2$, $r=4$ & $c(x) = \begin{cases} 0 & \text{if } x=0 \\ 1+\frac{1}{2}wrx^2 & \text{if } x>0 \end{cases}$, $P=2$

$$\pi(x) = TR(x) - TC(x)$$
$$= Px - (1+\tfrac{1}{2}wrx^2)$$
$$= 2x - (1+4x^2)$$

$$\Rightarrow \pi(x) = -4x^2+2x-1$$

$$\frac{d\pi(x)}{dx} = 0 \Rightarrow -8x+2=0 \Rightarrow x^* = 1/4$$

when $x^* = 1/4$, $\pi(x=1/4) = -1/4 + 1/2 - 1 = -3/4$, however, $\pi(x=0) = 0$!

$\therefore$ $x^* = 0$

(ii) Now $P=8$, so $\pi(x) = -4x^2+8x-1$

$$\frac{d\pi(x)}{dx} = 0 \Rightarrow -8x+8=0 \Rightarrow x^* = 1$$

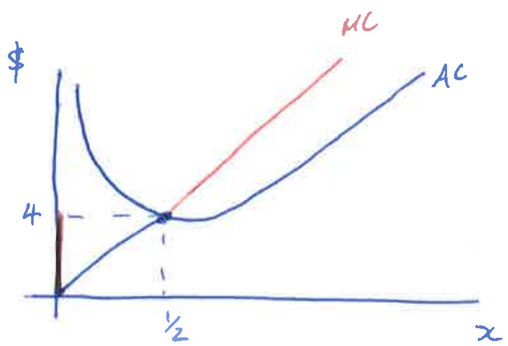
when $x^*=1$, $\pi(x=1) = 8-4-1 = 3 > 0$!

$\therefore$ $x^* = 1$

(iii) Recall, $c(x) = 1+4x^2$

$$AC(x) = \frac{c(x)}{x} = \frac{1}{x} + 4x \quad (1)$$

$$MC(x) = \frac{dc(x)}{dx} = 8x \quad (2)$$



Break even point occurs when $AC(x) = MC(x)$:

$$\Rightarrow \frac{1}{x} + 4x = 8x$$

$1 - 4x^2 = 0 \Rightarrow x^2 = 1/4 \Rightarrow x^{BE} = 1/2$; when $x = 1/2$, $MC(x=1/2) = 4$

$\therefore$ $P^{BE} = 4$ & $x^{BE} = 1/2$

(iv) Supply curve is as follows:

$$x(P) = \begin{cases} 0 & \text{if } P<4 \\ P/8 & \text{if } P \geq 4 \end{cases}$$ because $P = MC(x) = 8x$

(v) If w decreases, AC shifts downward, and MC rotates downward.

The portion of the supply curve above the break even point moves outward! (price goes down, output goes up)

Go through numerical example

Question 3

True, $\pi_{max} \Rightarrow C_{min}$ but $C_{min} \not\Rightarrow \pi_{max}$!

Think of Question 2 part (i).

To minimize cost the firm needed to produce $\frac{1}{4}$ units, however, this was not π_{max} !

π_{max} was to produce nothing! [This is a bit questionable]

Cost minimization = Best way to produce a certain level of output.

Profit maximization = Output level which attains highest income.

Could have a situation when $C_{min} \Rightarrow \overset{*}{x} > 0$ & $\pi_{max} \Rightarrow \overset{*}{x} = 0$

Tutorial 8

Question 1

(i) We have $f(\ell, k) = \ell^{1/4} k^{1/4}$ & $\bar{k} = 81$

In short run: $f(\ell) = \ell^{1/4}(81)^{1/4} = 3\ell^{1/4} \Rightarrow x = 3\ell^{1/4}$ (SR production function)

(ii) Recall $f(\ell) = x = 3\ell^{1/4}$

Solve for $\ell(x)$: $\ell^{1/4} = \frac{x}{3} \Rightarrow \ell = \frac{x^4}{81}$ (Demand for labour in SR)

(iii) In the short run: $c(x) = \bar{k}r + w\ell(x)$

We know $\bar{k} = 81$ & $\ell = \frac{x^4}{81}$, $\therefore c(x) = 81r + w\frac{x^4}{81}$ (short run cost function)

(iv) Let $x = 3$, $r = 1$, $w = 9$.

$C = 81 \cdot 1 + 9 \cdot \frac{3^4}{81} = 90$

Or

Let $x = 2$, $r = 1$, $w = 9$

$C = 81 \cdot 1 + 9 \cdot \frac{2^4}{81} = 82.77$

(v) Recall, in SR $c(x) = 81r + w\frac{x^4}{81} = FC + VC(x)$

$AVC(x) = \frac{VC(x)}{x} = w\frac{x^3}{81}$; $MC(x) = \frac{dc(x)}{dx} = 4w\frac{x^3}{81}$

Break-even point occurs when $AVC(x) = MC(x)$

When $w\frac{x^3}{81} = 4w\frac{x^3}{81} \Rightarrow x^* = 0$

$P_{SR} = AVC(x=0) = 0 \quad \therefore x_{SR} = P_{SR} = 0$!

(vi) Let $P = 12$. $\pi(x) = TR(x) - VC(x)$, $w = 9$, $r = 1$.

$\pi(x) = 12x - \frac{x^4}{9}$ $\Rightarrow$ Ignore FC in SR, FC is like a sunk cost!

F.O.C. x

$12 - \frac{4}{9}x^3 = 0 \Rightarrow x^3 = 27 \Rightarrow x^* = 3$

$\pi(x=3) = 12 \cdot 3 - \frac{3^4}{9} = 36 - 9 = 27 > 0 \quad \therefore x^* = 3$ is π_{max} in SR

(vii) In the long run $f(\tau,\kappa) = \tau^{1/4}\kappa^{1/4}$ & $C = 2x^2(wr)^{1/2}$

1. when $x=2$, $C_{SR} = 81r + w\dfrac{x^4}{81} = 81 + 9\cdot\dfrac{2^4}{81} = 82.77$

$C_{LR} = 2x^2(wr)^{1/2} = 2\cdot 2^2\cdot(1\cdot 9)^{1/2} = 24$ $\Rightarrow C_{LR} < C_{SR}$!

$\therefore$ LR costs are never larger than SR costs, in the long-run a firm is able to choose the least cost quantity of capital & labour. Whereas in the short run capital is fixed.

2. Let $w=9$, $r=1$, $P=12$. $C(x) = 2x^2(wr)^{1/2}$

$\therefore \pi(x) = 12x - 6x^2$

F.O.C. x

$12 - 12x = 0 \Rightarrow \overset{*}{x} = 1$, when $x=1$, $\pi(x=1) = 12-6 = 6 > 0$!

$\therefore \overset{*}{x}=1$ is π_{Max} !

3. LR economic profits appear to be smaller than SR economic profits. However, once you factor in the high fixed cost of capital in the SR, LR profits are actually higher.

(viii) Now suppose the government introduces a license fee = \$10.

1. $C = \begin{cases} 0 & \text{if } x=0 \\ 10 + 2x^2(wr)^{1/2} & \text{if } x>0 \end{cases}$

2. Optimal level of production has not changed, i.e. $\overset{*}{x}=1$. They can verify this!

$\pi(x=1) = 12 - 10 - 6 = -4 < 0$! $\therefore$ Now $\overset{*}{x}=0$ instead of 1.

Tutorial 9

Question 1

a) Demand: $x_J + x_M = 10 - p + 10 - 2p = 20 - 3p$

Supply: $x_A + x_B = 5 + p + 5 + 2p = 10 + 3p$

When Demand = Supply $\Rightarrow 20 - 3p = 10 + 3p$

$$6p = 10$$

$$p = 5/3 \; ; \; X = 15$$

b) Not enough information to answer! we need information on LR costs for both firms as well as the exit price.

Question 2 [Maybe skip this]

a) Long run costs are $C_{LR}(x) = 10x - 5x^2 + x^3$.

$AC(x) = 10 - 5x + x^2$; $MC(x) = 10 - 10x + 3x^2$

To find break even point we do the following:

$AC(x) = MC(x) \Rightarrow 10 - 5x + x^2 = 10 - 10x + 3x^2$

$$2x^2 - 5x = 0$$

$$x(2x-5) = 0$$

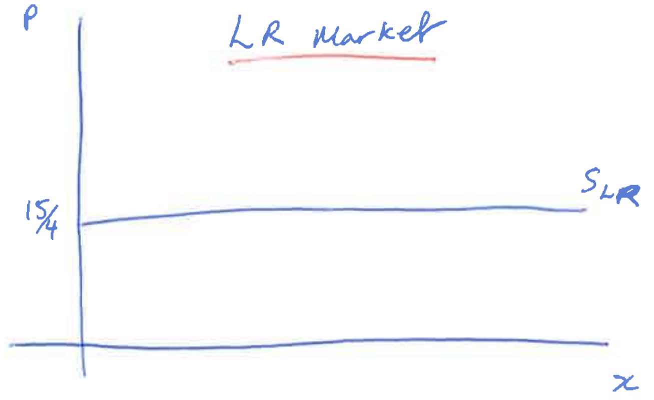
$\therefore x = 0$ or $5/2$! $\Rightarrow x^* = 5/2$

When $x^* = 5/2$; $AC(x = 5/2) = 10 - 5 \cdot 5/2 + (5/2)^2 = 15/4$

$\therefore$ $x^* = 5/2$ & $p^* = 15/4$ in the long run!

b)

LR Market



c) Market Demand: $x = 20 - 4p$

LR Market Supply: $p = 15/4$

Equilibrium Price: $p = 15/4$

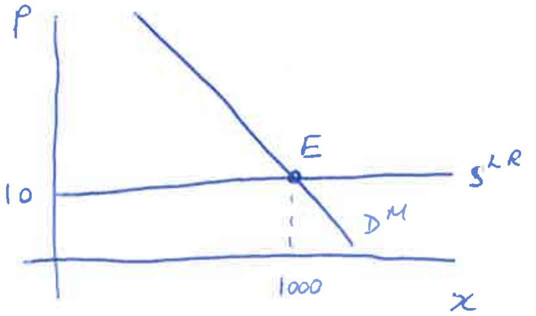
Equilibrium Quantity:

$x = 20 - 4 \cdot 15/4 = 5$ units

Question 3

a) Yes, a market is LR equilibrium if there is no entry/exit of firms. When $P = \$10 = AC_{LR} \Rightarrow \pi_{LR} = 0$. Hence, no entry/exit of firms.

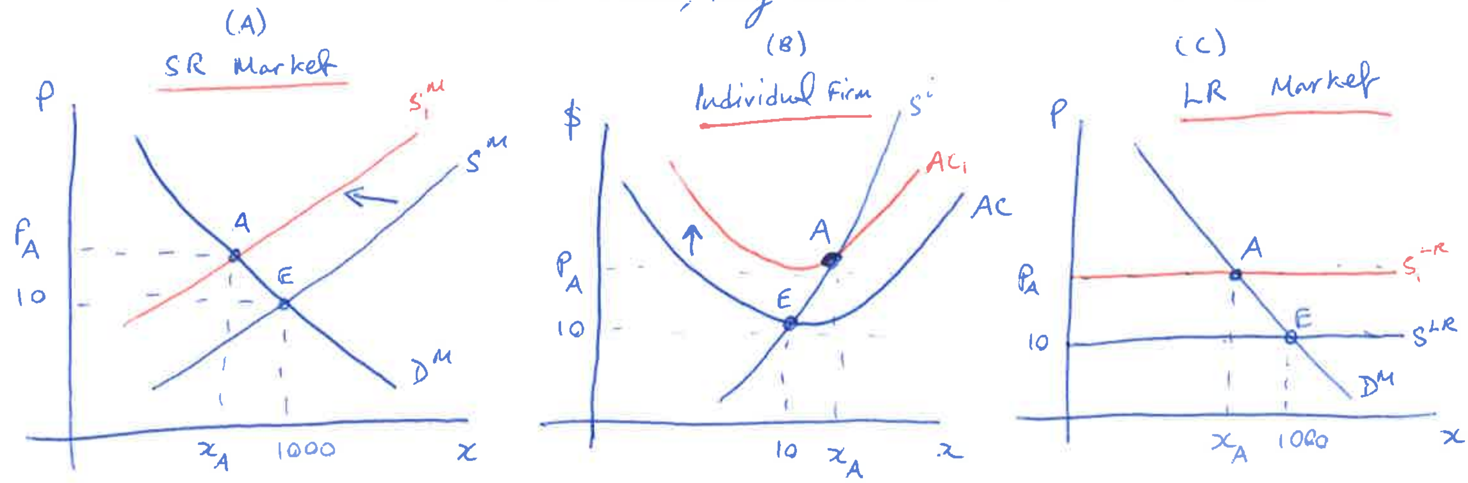
b)



Same Price & Quantity as in panel A!

c) SR economic profits are positive. At E, $P = \$10$ and the firm is breaking even in the long-run. However, $C_{SR} < C_{LR}$, hence, if $R - C_{LR} = 0$, then $R - C_{SR} > 0$! In SR capital costs are considered sunk, they don't count as economic costs.

d)
(i)



The licence fee is a fixed cost that is variable in the long run. Hence, in panel B AC shifts up to AC_1.

(ii) AC_{SR} & MC_{SR} are not affected by the licence fee, hence, there is no change in the SR equilibrium market, i.e. point E in panel A.

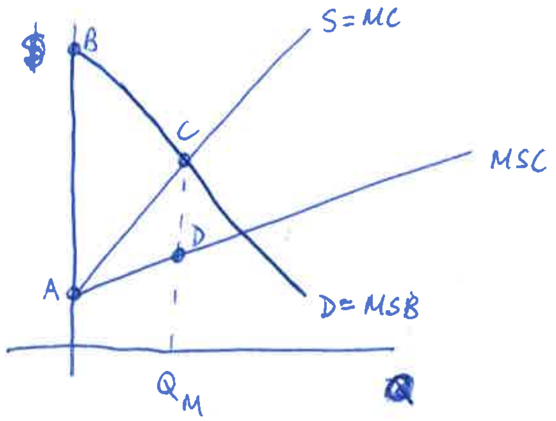
(iii) AC_{LR} has shifted up in panel B, hence, exit price has increased. At $P=10$, profit is negative. Firms will continue to exit until a new equilibrium price is reached, i.e. point A in panel B, and C.

(iv) We can see that equilibrium quantity has decreased (Panel C), however, each firm is producing more (panel B). Hence, the number of firms in the market must be less!

Tutorial 10

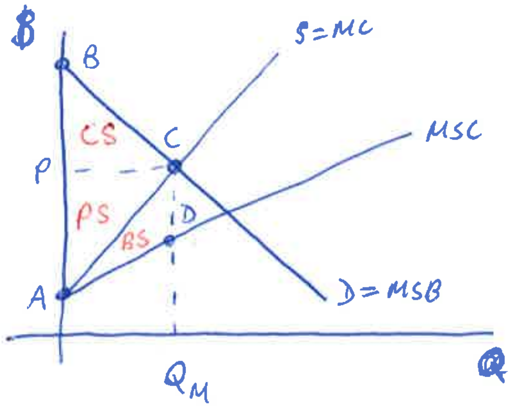
Question 1

(i)



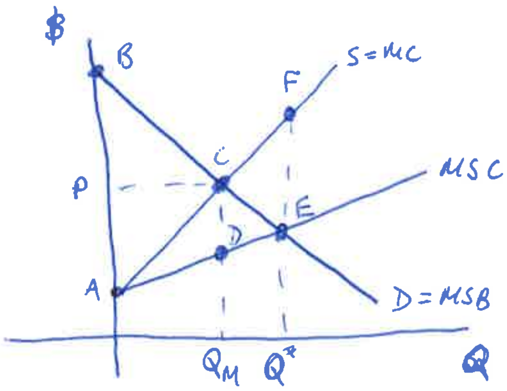
- Competitive market equilibrium is Q_M.
- Total surplus is area between MSB & MSC up to Q_M ⇒ ABCD.
- Do not need price to work out total surplus.

(ii)



- Consumer Surplus = PBC
- Producer Surplus = APC
- Bystanders' surplus = ACD

(iii)



- Social planner would decide to produce $\overset{*}{Q}$, where MSC = MSB.
- Total surplus = ABE

(iv) We know at Q^*, Bystanders Surplus = AEF (difference between MC & MSC up to Q^*).

CS + PS = ABC − CFE (difference between D & S up to Q^*).

However, we do not know the exact size of CS & PS as we do not know $\hat{P}$. The social planner is not concerned with price.

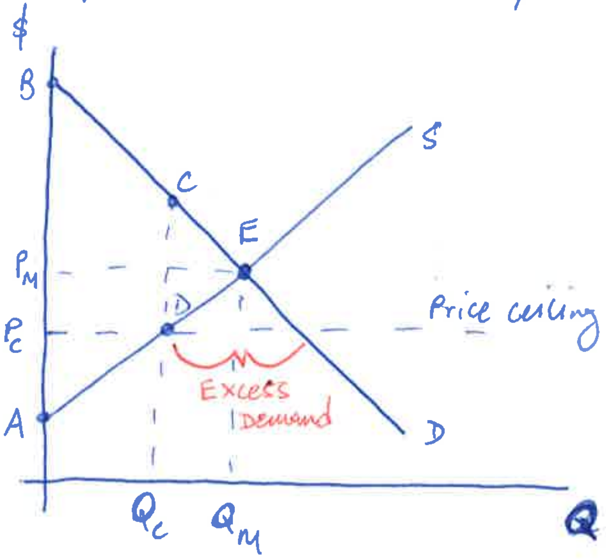
Question 2

1^{st} Welfare Theorem : Under certain conditions, competitive markets achieve efficient outcomes, i.e. outcomes that maximize total surplus.

Necessary Conditions :

1. There are no externalities
2. There are no policy-induced price distortions
3. Everyone acts as a "price taker"
4. There is no asymmetric information between market participants.

Suppose we have the following:

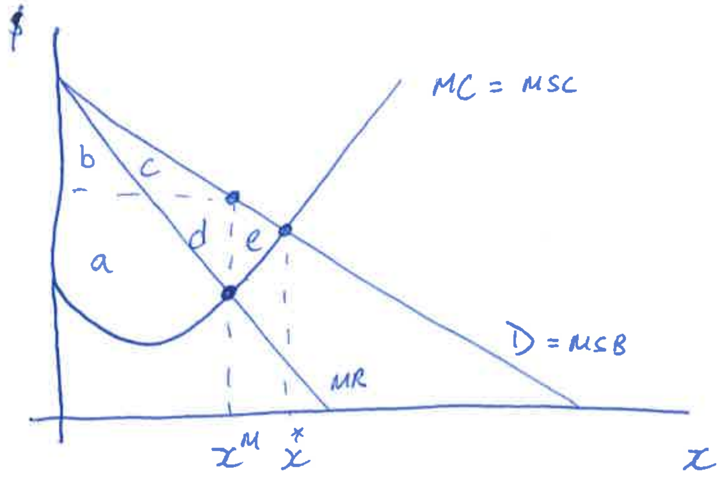


- Without a price ceiling the competitive market reached point E.
- When Q_M is produced $TS = ABE$.
- With the price ceiling P_c, Q_c is produced & $TS = ABCD < ABE$.
- Hence, the price ceiling reduces efficiency by CDE.

(*) The assumption that is violated is that there should be no price distortions.

Question 3

(i) We have the following:

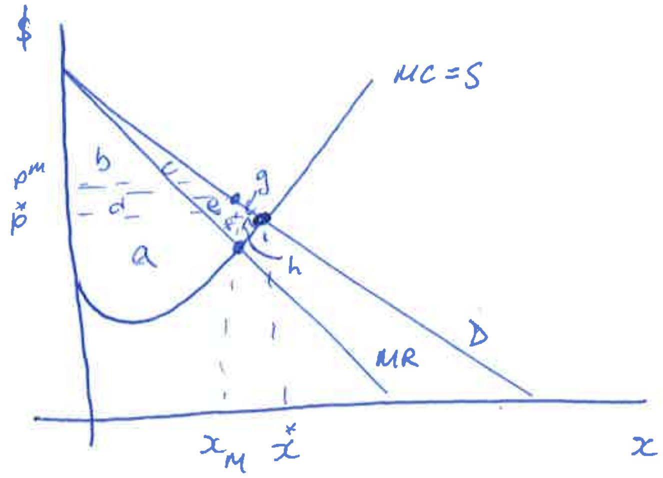


- Monopolist will set $MR = MC \Rightarrow x^M$!
- $TS = a+b+c+d$

(ii) A social planner will set $MSB = MSC \Rightarrow \overset{*}{x}$!

$\therefore$ $TS = a+b+c+d+e$

(iii)



- In a perfectly competitive market $S = D \Rightarrow \overset{*}{x}$ will be produced, and price will be $\overset{*}{p}$.
- Total surplus $= a+b+c+d+e+f+g+h$, i.e. same as with a social planner.

(iv) A social planner is not concerned with price, just the allocation of resources. Price is essentially just a transfer payment between buyers & sellers.

Tutorial 11

Question 1

a) $EV(\text{Tennis}) = P_r(s) \cdot V(s) + P_r(\bar{s}) \cdot V(\bar{s}) = 0.2 \times 400 + 0.8 \times 25 = \$100$

b) we know $u(x) = \sqrt{x}$.

$EU(\text{Bank}) = \sqrt{81} = 9$

$EU(\text{Tennis}) = 0.2\sqrt{400} + 0.8\sqrt{25} = 8$

$\therefore$ $EU(\text{Bank}) > EU(\text{Tennis}) \Rightarrow$ Lisa will work at the bank.

c) For certainty equivalent the following must hold:

$$u(x_{CE}) = EU(\text{Tennis})$$

$$\Rightarrow \sqrt{x_{CE}} = 8$$

$$\Rightarrow x_{CE} = 64$$

d) Risk Premium $= EV(\text{Tennis}) - x_{CE} = 100 - 64 = \$36$

e) we know Lisa is risk averse as $EV(\text{Tennis}) > EV(\text{Bank})$ but $EU(\text{Tennis}) < EU(\text{Bank})$. So we have $u(EV(\text{Tennis})) > EU(\text{Tennis}) \Rightarrow 10 > 8$.

Alternatively, we can tell because the risk premium is positive.

f) (i) $EV(\text{Insurance}) = 0.2(400-90) + 0.8(25+100-90) = \$90$

(ii) $EV(\text{No Insurance}) = EV(\text{Tennis}) = \$100 \Rightarrow EV(\text{No Insurance}) \neq EV(\text{Insurance})$

$\therefore$ Insurance is not actuarially fair.

(iii) $EU(\text{Insurance}) = 0.2\sqrt{400-90} + 0.8\sqrt{25+100-90} = 8.25 > EU(\text{No Insurance})$

$\therefore$ Lisa will pay for Insurance.

(iv) Firstly we want, $400 - P = 25 + b - P$

$$\Rightarrow b^* = 375$$

we also want $EV(\text{Insurance}) = EV(\text{No Insurance})$

$$\Rightarrow 0.2(400 - P) + 0.8(375 + 25 - P) = 100$$

$$\Rightarrow 400 - P = 100$$

$$P^* = 300$$

Question 2 (Extra Question)

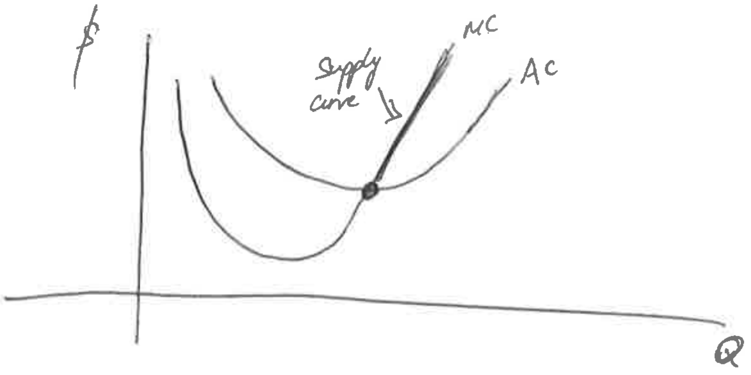
a) $EV(\text{Invest}) = 0.5 \times 100 \times 1.1 + 0.3 \times 100 + 0.2 \times 100 \times 0.9 = \$103$

b) No, we need to know about her risk preferences.

c) If Sophia is risk neutral, she will invest as $EV(\text{Invest}) > EV(\text{Not Invest})$.

d) If Sophia is risk loving, she will invest as $EV(\text{Invest}) > EV(\text{Not Invest})$, and investing involves more risk, which she likes.

e) If Sophia is risk averse, we cannot say for sure what she will do.



$$\pi = P \cdot Q - C(Q)$$

For profit max $\Rightarrow \frac{d\pi}{dQ} = 0 \Rightarrow P - c'(Q) = 0$

$$\Rightarrow P = MC(Q) \quad \text{[Profit maximizing condition]} \quad (1)$$

A firms profit is:

$$\pi = PQ - C(Q) = P \cdot Q - \frac{C(Q)}{Q} \cdot Q$$

$$= P \cdot Q - AC(Q) \cdot Q$$

$$\Rightarrow \pi = [P - AC(Q)]Q \quad (2)$$

Sub (1) into (2):

$$\pi = [MC(Q) - AC(Q)]Q$$

When $MC(Q) > AC(Q) \Rightarrow \pi > 0$

When $MC(Q) = AC(Q) \Rightarrow \pi = 0$

When $MC(Q) < AC(Q) \Rightarrow \pi < 0$

$\Rightarrow \therefore$ Firm will only supply when $\pi > 0 \Rightarrow MC(Q) > AC(Q)$

$$\min_{\{Q\}} AC(Q) \Rightarrow \min_{\{Q\}} \frac{C(Q)}{Q}$$

$u = C(Q) \qquad V = Q$

$u' = C'(Q) \qquad V' = 1$

$$\Rightarrow \frac{C'(Q) \cdot Q - C(Q)}{Q^2} = 0$$

$$\frac{C'(Q)}{Q} = \frac{C(Q)}{Q^2}$$

$$C'(Q) = \frac{C(Q)}{Q}$$

$$\Rightarrow MC(Q) = AC(Q)$$

TUTORIAL 10 – Welfare

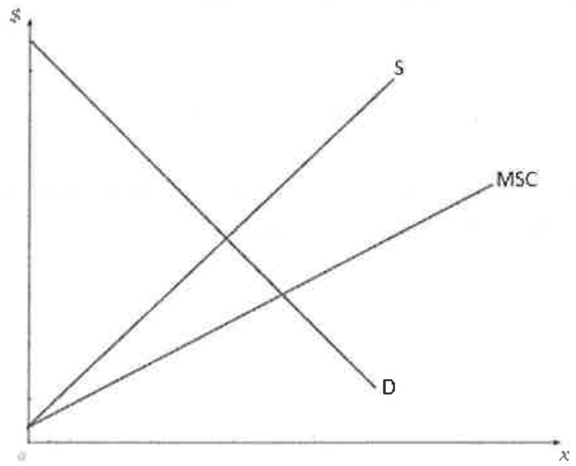
Intermediate Microeconomics, Spring 2017

Massimo Scotti

Understanding consumer surplus, producer surplus, total surplus for the society and the idea of efficiency by looking at cases in which the First Theorem of Welfare fails.

Question 1

The following graph represents a competitive market with a positive externality in production (there are no other externalities and there are no price distortions).



i. What is the total surplus generated by the competitive market?

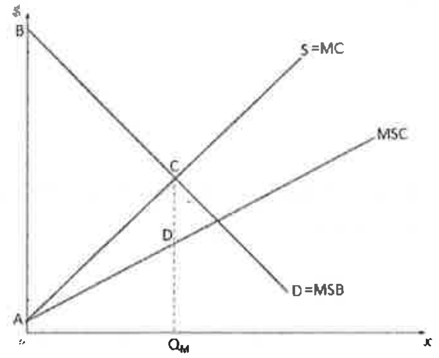
ii. Identify how the total surplus generated by the competitive market is split between consumer surplus, producer surplus and bystanders' surplus.

iii. What would be the surplus generated by a benevolent and omniscient social planner?

iv. What is consumer surplus, producer surplus and bystanders' surplus in the case of the social planner's solution?

Answer:

i.

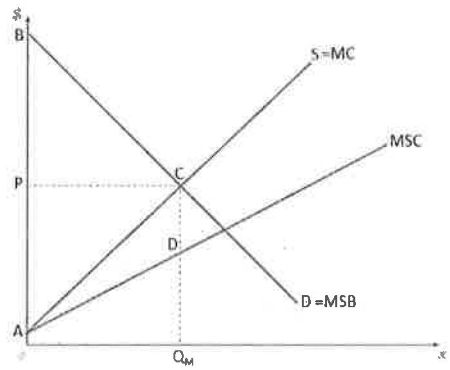


To find total surplus, you have to identify the MSC curve, the MSB curve and the quantity produced. The MSC curve is given by the exercise. The MSB curve is the D curve (because we are told that there are no externalities in consumption). The quantity produced by the market is Q_M. Then, total surplus is the area comprised between the MSC curve and the MSB curve up to Q_M, that is:

Total surplus generated by the market= area ABCD

Note that you do not need to know the price to find total surplus (what you need is just the quantity produced and MSC and MSB curves).

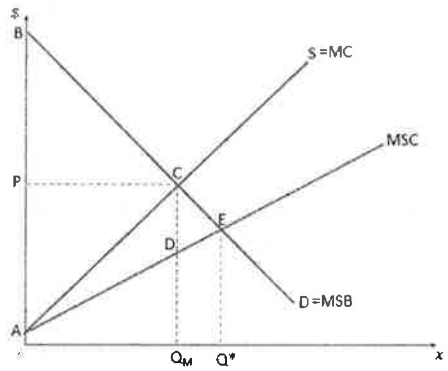
ii.



To know the total surplus generated by the market is split you bring in the price. The equilibrium price is P. Thus:

Consumer surplus=PBC
Producer surplus=APC
Bystanders' surplus=ACD

iii.

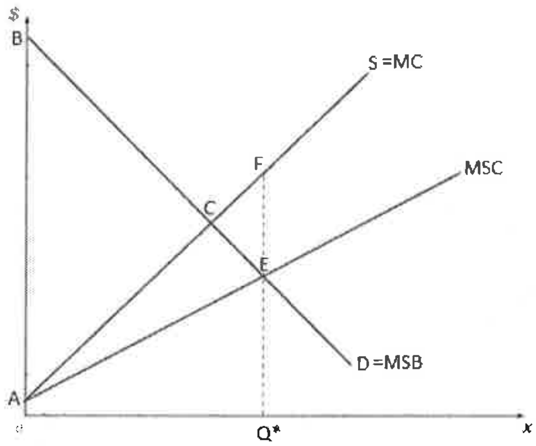


The social planner would decide to produce Q* where MSC=MSB. Thus:

Total surplus generated by social planner: ABE

So, by letting this market with a positive externality in production free to operate we have a loss in surplus (relative to what we could potentially achieve) equal to the area DCE.

iv. Let's start from the surplus of bystanders. Since the social planner is going to produce Q*, there is no doubt that the surplus of bystanders is going to be equal to area AEF (the difference between private and social cost).



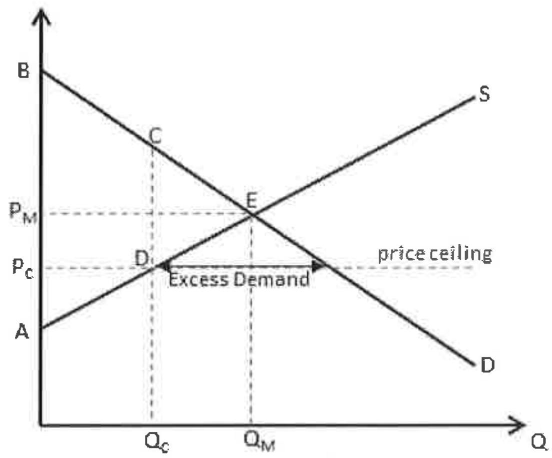
What about consumer and producers surplus? At Q*, the sum of producer surplus and consumer surplus is equal to areas ABC-CFE (the difference between private benefit, i.e. D, and private cost, i.e. S). However, we cannot tell how this surplus is split between consumers and producers because we do not have information about the price that the central planner is going to charge for each unit of the good that is produced. For example, if each unit produced was charged at a price equal to the cost of producing it, then the whole surplus (the entire area ABC-CEF) would go to the consumers (producers would strike zero profits since the price of each unit would be just enough to cover production cost of that unit). If instead the planner was charging for each unit a price equal to the marginal willingness to pay for that unit, than the whole surplus (the entire area ABC-CEF) would go to the producers. Furthermore, note that any "pricing" solution between these two extreme ones would be feasible (i.e. each unit could be sold at any price between the marginal cost of and the willingness to pay for that unit).

Question 2

Consider a competitive market with no externalities. With the aid of a graph explain why the introduction of a binding price ceiling would reduce efficiency. Which assumption of the 1st theorem of welfare is violated?

Answer:

Consider the graph below. Without a price ceiling the competitive market reached the equilibrium at point E. Quantity Q_M is produced and consumed. Since there are no externalities, the S and D curves represent the MSC and the MSB curves. Thus, total surplus for the society at the market equilibrium is equal to area ABE (where P_MBE is consumer surplus and P_MAE is producer surplus). If a price ceiling P_C is introduced, each unit must be sold at P_C. This creates an excess demand. Without a price ceiling, the price mechanism would reduce this excess demand to zero and restore equilibrium. The price ceiling prevents the price mechanism from functioning. Since each unit must be sold at P_C, producers would produce only quantity Q_C. The total surplus for the society at quantity Q_C is equal to area ABCD. Thus, the price ceiling reduces efficiency by creating a loss in surplus equal to area DCE.



(A side note: there is also a redistribution effect of the price ceiling with the producer surplus decreasing to AP_C D and the consumer surplus increasing to P_CBCD).
The assumption that is violated is that there should not be price distortions.

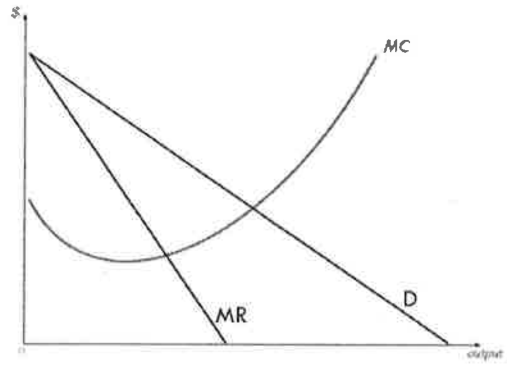
Question 3

Consider the following graph representing a monopolistic market. Assume that there are no externalities, no policy-induced price distortions and information is symmetric. Is this market efficient? To answer this question, find:

i. The total surplus of the society at the market equilibrium

ii. The total surplus that would be generated by a benevolent and omniscient social planner.

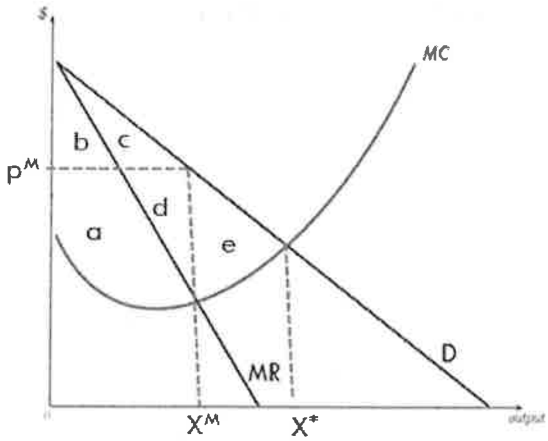
iii. What can you say about the price charged by the social planner?



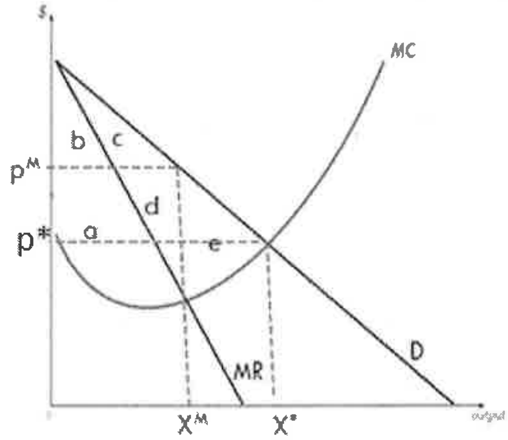
Answer

i. The monoplosit produces quantity X^M where MR=MC (this is the quantity that maximizes his profits). The price he choose is P^M (given the market demand he faces, that is the highest per unit price he can charge in order to sell X^M). Since there are no externalities and no policy-induced price distortions, the MC curve of the monopolist is also the MSC curve. And the D curve is also the MSB curve. Thus, total surplus for the society at the market equilibrium is equal to area a+b+c+d.

ii. A social planner produces where the MSC is equal to the MSB. Thus, he will produce X*. At X* the total surplus for the society is equal to area a+b+c+d+e. Thus, a monopolistic market is not efficient because it generates a smaller surplus relative to what a benevolent and omniscient planner would do. In particular the loss in surplus is equal to area e.



iii. If the market were competitive, the equilibrium would be were S intersects D. What is S? It is the MC curve of the monopolist. Hence the equilibrium would be at price p* and quantity X*. Note that in a competitive equilibrium, output is the same as the one chosen by the social planner, that is, the competitive equilibrium would be fully efficient.



iv. Prices are irrelevant when it comes to a central planner deciding how much to produce and how to allocate production. The problem solved by the planner is a pure allocation problem. The planner decides which unit to produce and to whom to allocate that one unit, making it sure that each unit is produced at the lowest cost for society and then it is allocated to the consumer that values it most. Essentially, prices become irrelevant because the role that is performed by prices in a decentralized economy (i.e. competitive market) is now performed by the central planner.

If you really want to bring prices in, note that they will simply affect how surplus is distributed. The planner could charge each unit differently, setting a price between the MSC and the MSB of each unit. The price of each particular unit will then simply affect how the surplus on each unit is split between the producer and the consumer of that one unit.

Note that among these pricing solutions there is, of course, also the one that would arise if the market were competitive. That is the planner could choose a price that is the same for each unit and equal to p*.

Question 1

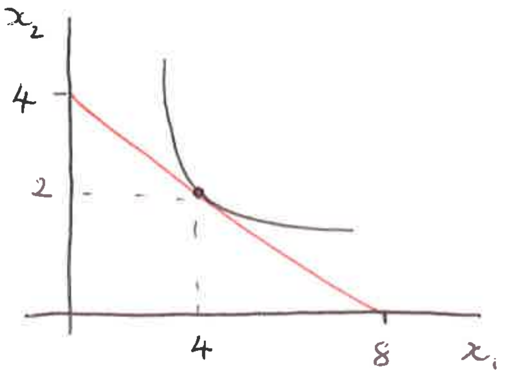
$u(x_1, x_2) = x_1 x_2$; $p_1 = 3$; $p_2 = 6$; $I = 24$

a) opportunity cost of good $1 = \frac{p_1}{p_2} = \frac{3}{6} = \underline{\frac{1}{2}}$

b) $MRS = -\frac{MU_1}{MU_2} = -\frac{x_2}{x_1}$

when $(x_1, x_2) = (12, 4) \Rightarrow MRS = -\frac{4}{12} = \underline{-\frac{1}{3}}$

c) we have $3x_1 + 6x_2 = 24 \Rightarrow 6x_2 = 24 - 3x_1 \Rightarrow \underline{x_2 = 4 - \frac{1}{2}x_1}$



d) we know $MRS = -\frac{p_1}{p_2} \Rightarrow \frac{x_2}{x_1} = \frac{1}{2} \Rightarrow x_1 = 2x_2$ (1)

Recall, $3x_1 + 6x_2 = 24$ (2)

Sub (1) into (2): $6x_2 + 6x_2 = 24 \Rightarrow \boxed{x_2^* = 2 ;\ x_1^* = 4}$

e) see graph

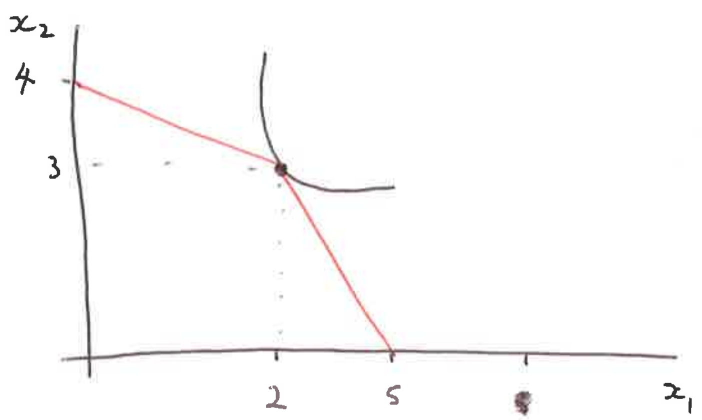
f) $\underline{u(4, 2) = 8}$

Question 2

Suppose $P_1 = 3$; $P_2 = 6$; $I = 24$

a) when $x_1 \leq 2$; $3x_1 + 6x_2 = 24 \Rightarrow x_2 = 4 - \frac{1}{2}x_1$ (1)

when $x_1 > 2$; $6x_1 + 6x_2 = 24 - 6 \Rightarrow x_2 = 3 - x_1$ (2)



when $x_2 = 0$, $x_1 = 3$

$\therefore x_1 = 3 + 2 = 5$

b) suppose $u(x_1, x_2) = x_1 x_2$

Assume no tax, we know $(x_1^*, x_2^*) = (4, 2)$

Cost $= 3 \times 2 + 6 \times 2 + 6 \times 2 = 30 > 24 \therefore$ not possible

we could have, $MRS = -\frac{x_2}{x_1}$ & $\frac{P_1}{P_2} = 1 \therefore x_2 = x_1$ (1)

Sub (1) into (2): $x_2 = 3 - x_2 \Rightarrow 2x_2 = 3 \Rightarrow x_2 = 3/2$; $x_1 = 3/2 + 2 = 7/2$

$u(7/2, 3/2) = 21/4 = 5.25$

At the kink, $u(2,3) = 6 > 5.25 \therefore$ optimal bundle is at the kink!

$\therefore$ optimal bundle $= (2,3)$

Answer should be $(x_1, x_2) = (5/2, 5/2)$

Question 3

Suppose $f(\ell, k) = \ell k$; $r = 5$; $w = 10$

a) Suppose $\bar{k} = 5$ & $x = 50 \Rightarrow \ell \bar{k} = 50 \Rightarrow \ell = 10$

$C_{SR} = w\ell = \underline{\underline{100}}$ (economic cost)

b) In the long run:

$$\min_{\{\ell, k\}} 10\ell + 5k$$

s.t.

$\ell k = 50$ (2)

$TRS = -\frac{w}{r} \Rightarrow -\frac{k}{\ell} = -\frac{10}{5} \Rightarrow k = 2\ell$ (1)

sub (1) into (2):

$\ell \cdot 2\ell = 50 \Rightarrow \ell^2 = \frac{50}{2} \Rightarrow \underline{\ell^* = 5 ;\ k^* = 10}$

$\therefore$ In the long run the firm will choose $k^* = 10$ instead of $\bar{k} = 5$!

Question 4

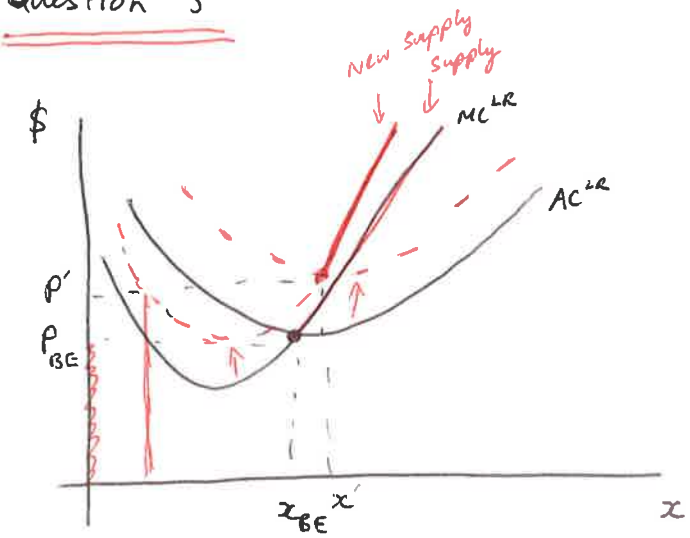
We know $C(x) = 100 + x^2$

$AVC(x) = x$ & $MC(x) = 2x$

when $AVC(x) = MC(x) \Rightarrow x = 2x \Rightarrow x^*_{SD} = 0$; $p^*_{SD} = 0$

We know $p_E = 10 > p_{SD}$ $\therefore$ <u>Firm should produce in SR!</u>

Question 5



c) supply curve shifts to the left after wage increase.

d) Suppose $C(x) = 100 + x^2$

$$AC(x) = \frac{100}{x} + x \; ; \; MC(x) = 2x$$

$$AC(x) = MC(x) \Rightarrow \frac{100}{x} + x = 2x$$

$$x = \frac{100}{x}$$

$$x^* = 10 \; ; \; p^*_{BE} = 20$$

Question 6

a) $EV_\beta = \frac{1}{2} \times 10 + \frac{1}{2} \times 4 = 7$; $EV_\alpha = 7$

b) Suppose $u(x) = x^2$

$$EU_\beta = \frac{1}{2} \times 10^2 + \frac{1}{2} \times 4^2 = 58 \; ; \; EU_\alpha = 7^2 = 49$$

So $EU_\beta > EU_\alpha \Rightarrow$ Sophia will choose β

c) Sophia is risk loving. She prefers the gamble over the certain outcome even though they both have the same expected value.