Spill-overs in the inter-generational belief formation

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Approximating Common Knowledge with Common Belief

Monderer and Samet (1989)

- Games assume common knowledge of all elements of the game
  - Is this always true?
  - Does it matter?
- P-Belief is the generalization of Aumann's (1976) no-agreement theorem

  After several iterations of "I know that you know that I know . . . , '' one stops and ''he does not know that . . . '' from then on. But certainly in this case he may still "believe that . . . " with some certainty. And then he may believe that the other does, and so on.
- 1-belief = common knowledge = common believe
- The main motivation for the experiment
  - Can almost common knowledge work?

Real world examples with multiple equilibria

belief is a “traffic light”

- Big push theory
  - Inefficiency of small investments
- O-Ring theory
  - “Challenger shuttle disaster”
- Currency crises models
  - All three generations of models say the same
- Bank runs
- Continental Airlines
- Social norms

Is there a game theoretical approximation to these examples?

Game with multiple equilibria

Van Huyck et al (1990)

- Minimum effort game
  - Note similarities to the public good game and level of uncertainty
- $e \in \{1,2,\dots,\bar{e}\}$
- $\pi\left(e_i, \underline{e_i}\right) = a\left[\min\left(e_i, \underline{e_i}\right)\right] - be_i,\ a > b > 0,$
- $\underline{e_i} = \min(e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_n)$
- Player's BR to $\underline{e_i}$ is to choose e_i equal to $\underline{e_i} \Leftrightarrow u_i(\underline{e_i}, \underline{e_i}) \geq u_i(\underline{e_i}, \underline{e_i})$
- $e_i \neq \underline{e_i} \Leftrightarrow$ disequilibria
- $\min(e_1, \dots, e_n) \neq \bar{e} \Leftrightarrow$ multiple equilibria

A Role of Minimum

Van Huyck et al (1990)

- n-tuple $(\bar{e}, \ldots, \bar{e})$ is a PDEP (payoff-dominant equilibrium point)
- If $F(e_j)$ as cdf for a player's action then PDEP is defined as

$F(\bar{e}) = 1$

$F(e_j) = 0$ if $e_j < \bar{e}$

- Yet if $e_1, \ldots, e_n$ i.i.d. then $F_{\min}(e) = 1 - [1 - F(e_j)]^n$ then PDEP is defined as

$F_{\min}(\bar{e}) = 1$

$F_{\min}(e) = 0$ if $e < \bar{e}$

- Suppose that a player is uncertain that the n-1 players will select the payoff-dominant action $\bar{e}$,

e.g. $F(1) = \varepsilon$ then

If $n \to \infty \Rightarrow F_{\min}(1) \to 1$

- Even a remote possibility of deviation motivates defection

The Original Experiment

Van Huyck et al (1990)

Payoff Table in Van Huyck et al.'s (1990) Minimum Effort Game

		Smallest Value of X Chosen						
Your Choice of X		7	6	5	4	3	2	1
	7	*1.30*	1.10	0.90	0.70	0.50	0.30	0.10
	6	–	*1.20*	1.00	0.80	0.60	0.40	0.20
	5	–	–	*1.10*	0.90	0.70	0.50	0.30
	4	–	–	–	*1.00*	0.80	0.60	0.40
	3	–	–	–	–	*0.90*	0.70	0.50
	2	–	–	–	–	–	*0.80*	0.60
	1	–	–	–	–	–	–	*0.70*

$$\pi\left(e_i, \underline{e_i}\right) = k + a\left[\min\left(e_i, \underline{e_i}\right)\right] - be_i;\ k = 0.6;\ a = 0.2;\ b = 0.1$$

The Original Experiment

Van Huyck et al (1990)

Table 3

(b) Group Minima – Van Huyck et al. (1990) Experiments

	Rounds									
	1	2	3	4	5	6	7	8	9	10
Group										
1	2	2	2	1	1	1	1	1	1	1
2	2	1	1	1	1	1	1	1	1	1
3	4	2	2	1	1	1	1	1	1	1
4	4	2	3	1	1	1	1	1	1	1
5	3	2	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1

- Coordination failure
  - Few brave souls try a probing, note how Von Neumann's mini-max predicts the outcome
- Game needs coordination
  - What kind of device would be fun to chose to construct the efficient beliefs?

Inter-generational device

Chaudhuri et al (2009)

Table 2

(a) Experimental Design for the Inter-generational Games in Block I

No.	Game	Number of Generations	Rounds Per Generation	Subjects Per Generation	Number of Subjects
1	Replicator: No-Advice or History	4	10	8	32
2	Progenitor: No History or Advice, but Advice left	1	10	8	8
3	Private Advice-Plus-History	6	10	8	48
4	Private Advice Only	6	10	8	48
5	Public Advice	9	10	8	71
	(Public Advice *Not* Read Aloud – Almost Common Knowledge)	(5)	(10)	(8)	(40)
	(Public Advice Read Aloud – Common Knowledge)	(4)	(10)	(8)	(31)
	Total				207

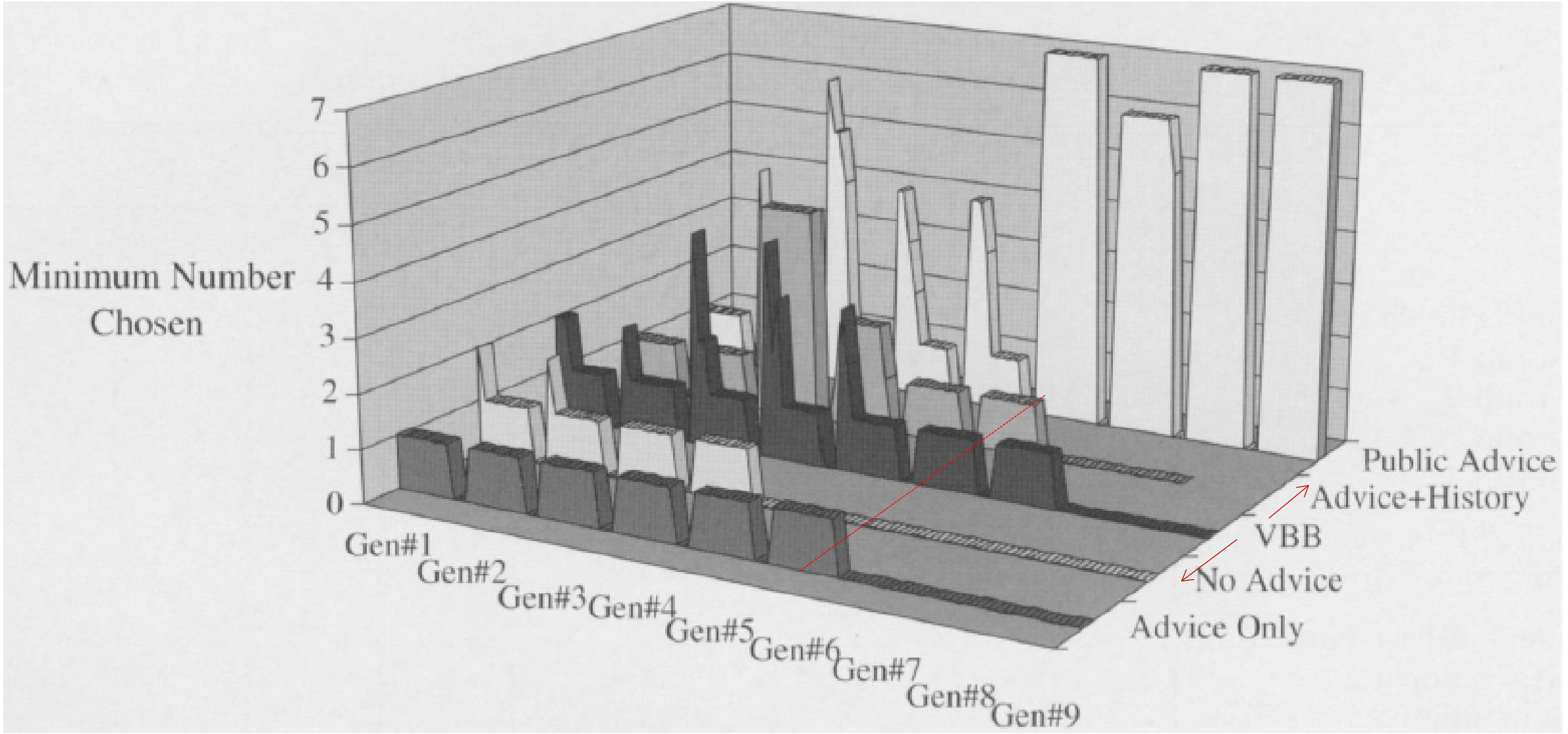


Fig. 1. *Behaviour of the Minimum Across Block I Games*

Does IIA holds?
N=1860?

Including women?

Table 5

Estimated Probabilities of Choosing a Particular Number in Block I Games
(at treatment averages of other variables)

Choice	Progenitor Generation	No Advice	Advice Only	Advice-Plus-History	Public Advice Almost Common Knowledge	Public Advice Common Knowledge
1	0.56	0.65	0.76	0.49	0.22	0.00
2	0.22	0.20	0.15	0.24	0.22	0.00
3	0.08	0.07	0.04	0.10	0.13	0.00
4	0.09	0.07	0.04	0.11	0.21	0.01
5	0.03	0.02	0.01	0.03	0.09	0.01
6	0.01	0.01	0.00	0.02	0.07	0.02
7	0.01	0.00	0.00	0.01	0.07	0.96

Controlling for Quality of Advice

Table 2

(b) Experimental Design for Block II Games

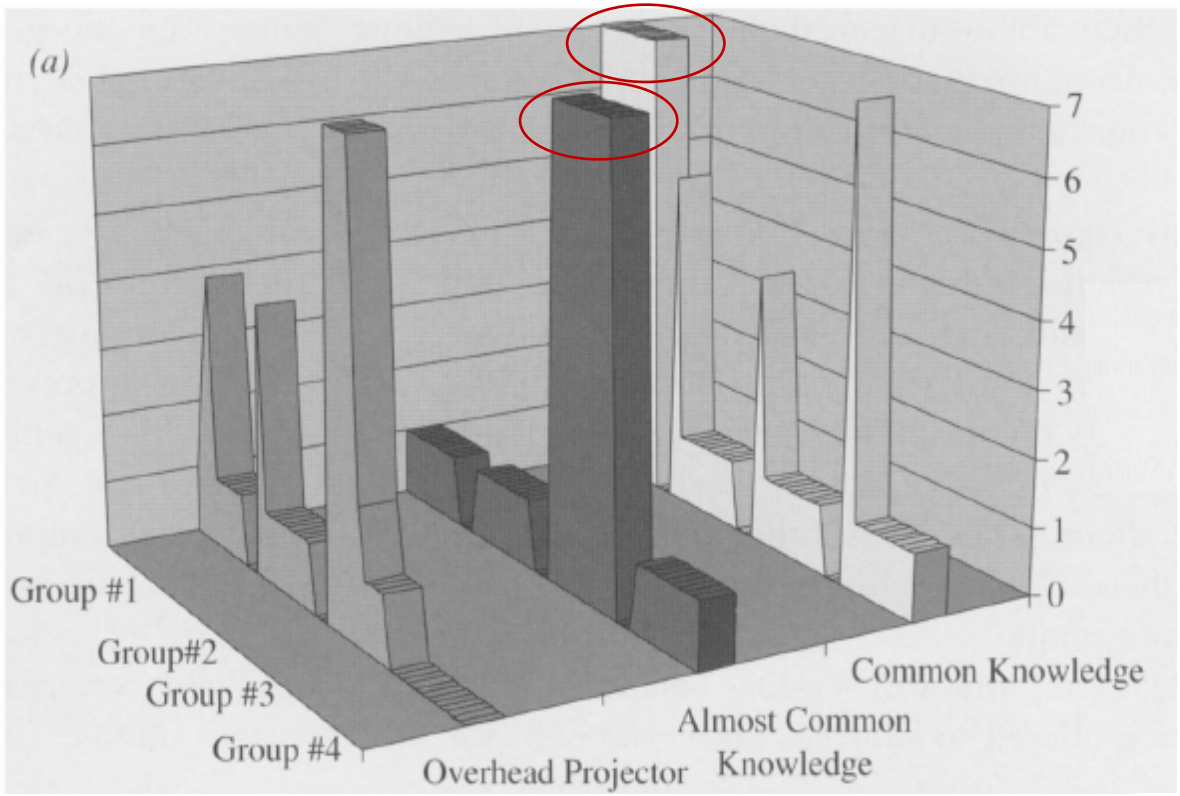
No.	Game	Number of Groups	Rounds Per Group	Subjects Per Group	Advice Quality	Number of Subjects
6	Common Knowledge of Advice	4	10	8	Good	32
7	Almost Common Knowledge of Advice	4	10	8	Good	32
8	Advice projected on Overheads for all subjects to see	3	10	8	Good	24
9	Common Knowledge of Advice	4	10	8	Very Good	32
10	Almost Common Knowledge of Advice	4	10	8	Very Good	32
11	Private Knowledge of Advice	3	10	8	Very Good	24
	Total					176

Table 7

The Exact Advice Given to the Subjects in the Block II Games

Subject	Good Advice	Very Good Advice
1	Be ahead of everybody, start with a 6, then go down to 5, etc. You will be able to make the most money that way.	Pick 7 every time, EVERY TIME. If everyone picks 7 every time, everyone will make the max per round $1.30×10 $13.00), plus you can make the full $1.28 for each of the predictions rounds. Don't be stupid. Pick 7. Honestly, you're here for the money anyway, right?
2	Choose #7. Don't be tempted to deviate but everyone must choose #7.	If you don't start the first round with '7' then the pattern thereafter will be '7' or lower. Bottom line – you must begin the first period with a '7'...Or else!!!!
3	True, if everyone selects #7, you have max profit. But when you see the smallest # move down, you should follow.	Pick 7 for crying out loud! But if there is a weirdo who picks lower, pick that number too. Pick 7!!! Trust each other it will help you too!
4	If *everyone* continues to pick 7 you will maximise your profit. Anything else and profit maximisation is not possible.	For the first round, you must trust the other participants & choose 7. Choosing 7 gives the maximum payoff. The (*sic*) adjust your choice by following the trend after the first round. Be consistent!
5	Start with 7! Everyone agree at least once. Once someone starts using *one* join them.	It would be best for everyone to choose 7 each time. However, if one person consistently chooses a lower number, you will make more profitably conforming to them.
6	Follow the trend. Ideally you want to maximise at 7 but inevitable someone doesn't get it.	Picking 7 will yield the maximum payoff pick 6 if everyone picks 7. So start out picking 7, however, some people are very untrusting and will or 5 – if this happens, 6, follow the trends, if everyone starts picking start

“Good” and “Very Good” treatments



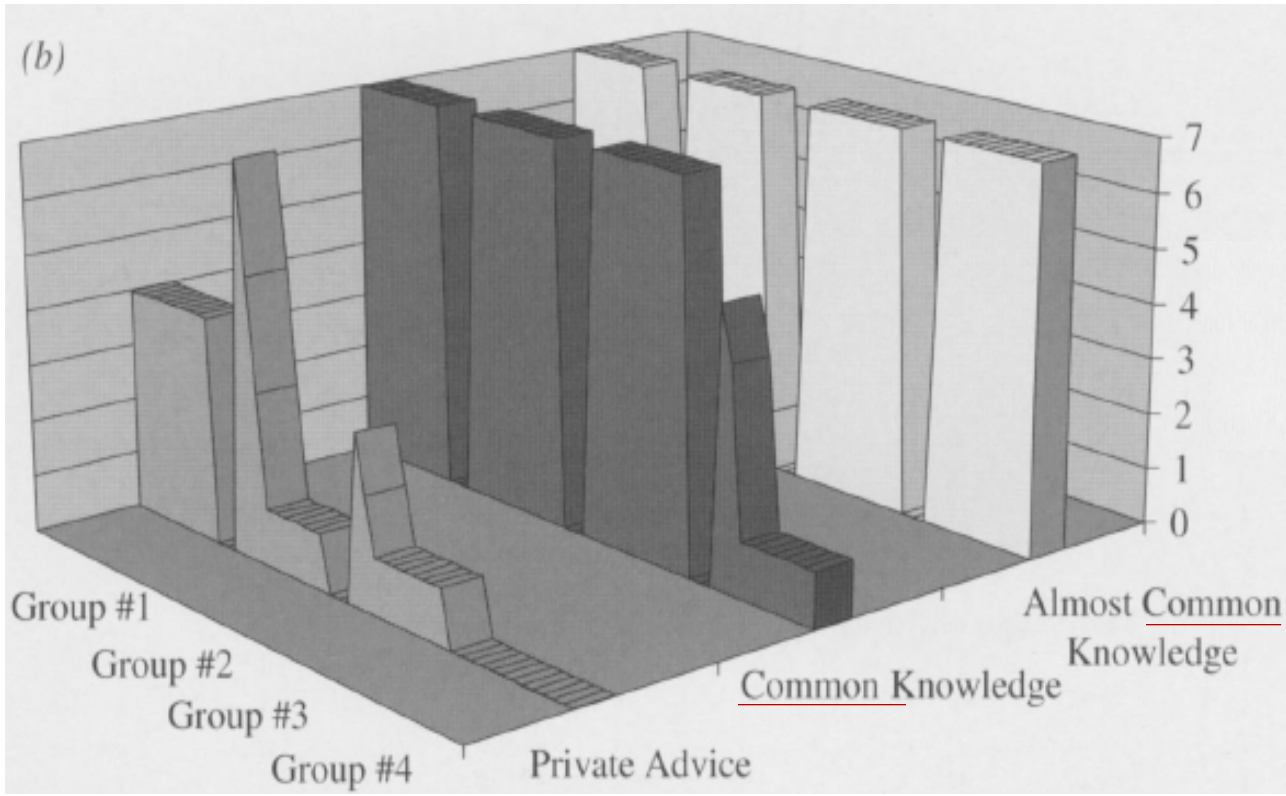


Fig. 2. *(a) Behaviour of the Minimum Across Block II Games with ‘Good’ Advice and (b) Behaviour of the Minimum Across Block II Games with ‘Very Good’ Advice*

Table 9

Probability of Observing a First Round Minimum of 1, 2, 3, 4, 5, 6 or 7 Across the Various Games in Block II

	Minimum						
Game	1	2	3	4	5	6	7
Private Knowledge and 'Very Good' Advice	0	0	0.33	0.39	0.105	0.14	0.03
Almost Common Knowledge and 'Very Good' Advice	0	0	0	0	0	0	1
Common Knowledge and 'Very Good' Advice	0	0	0	0	0.44	0	0.56
Almost Common Knowledge and 'Good' Advice	0.59	0	0	0	0.20	0	0.21
Advice on Overheads and 'Good' Advice	0.565	0	0	0.16	0.105	0.07	0.10
Common Knowledge and 'Good' Advice	0	0	0	0.44	0.15	0.11	0.295

Conclusion 1:

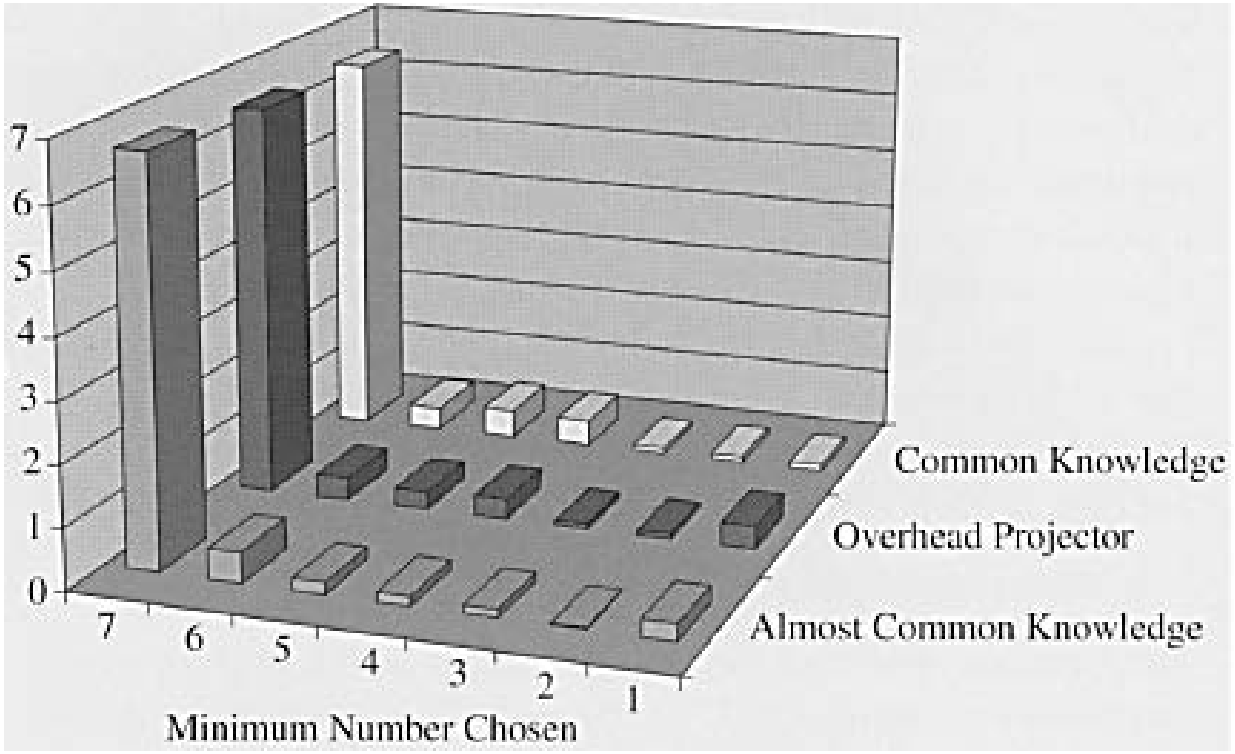
On the basis of our Block II results, we can say that if advice is strong enough (with all subjects in a group strongly exhorting their successors to choose 7) then efficient coordination is achieved regardless of the manner in which the advice is distributed, as long as it is public.

Conclusion 2:

Also on the basis of our Block II results we find that when advice is insufficiently strong, then efficient coordination is likely to be established only if that advice is distributed as common knowledge.

Block II Beliefs and Common p-beliefs

distributions of independently elicited first order beliefs



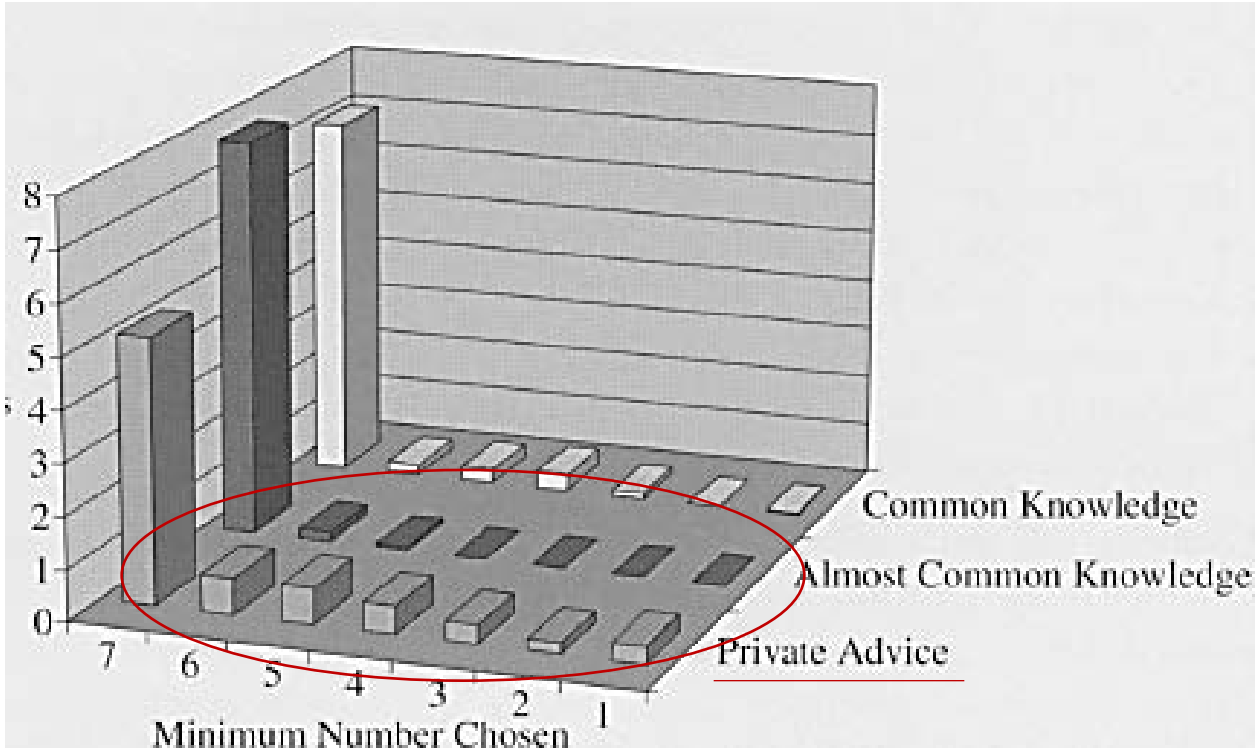


Fig. 4. *(a) Comparison of Beliefs Across Block II Games with 'Good' Advice and (b) Comparison of Beliefs Across Block II Games with 'Very Good' Advice*

Controlling for Quality of Advice

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(b) Experimental Design for Block II Games

No.	Game	Number of Groups	Rounds Per Group	Subjects Per Group	Advice Quality	Number of Subjects
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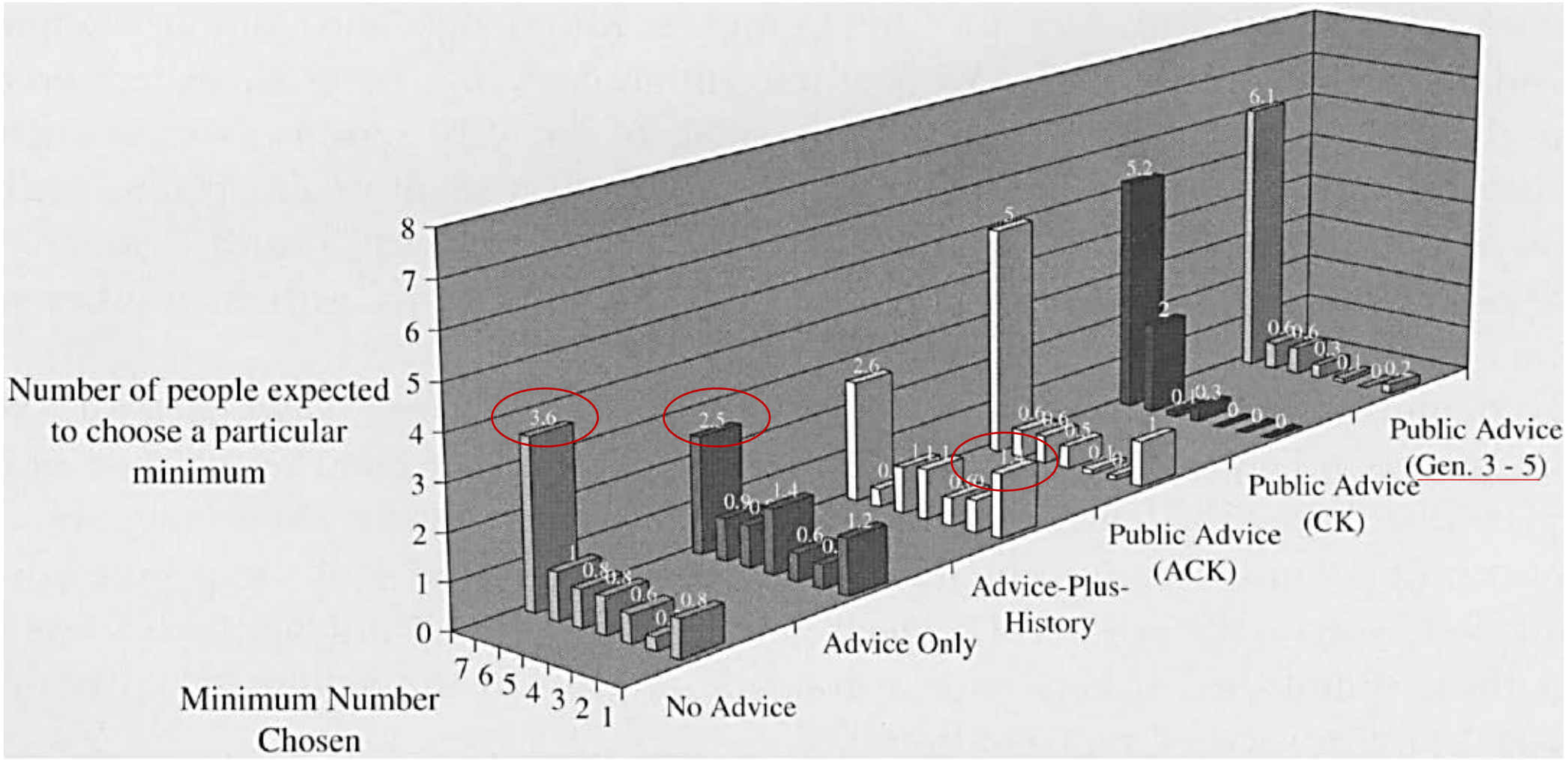

Fig. 5. *Comparison of Beliefs Across Block I Games*

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Chaudhuri et al (2009)

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Bottom line

- Conclusions
  - Private advice did the opposite of what was expected
    - Regardless of the quality
  - Public advice has to be of high quality
- Some personal notes
  - Inspired by Chew's "Rational Ritual: Culture, Coordination and Common Knowledge" (2001)
    - But is this "mapping" accurate?
      - We have dome resembles of a mathematic abstraction to some real world example
      - Is that enough?
    - Are loud words truly justified?
    - Imagine that game does not have Block I
      - How weird would that be?